SOLUTIONS => 6.5 Solving Quadratic Equations by Factoring

1.

a) $x^2 - 11x + 28 = 0$ (A: -11, M: 28)

$-4 \times -7 = 28$

$-4 + -7 = -11$

$(x-4)(x-7)=0$

$x-4=0$ or $x-7=0$

$x=4$ $\quad x=7$

b) $x^2 - 7x - 30 = 0$ (A: -7, M: -30)

$3 \times -10 = -30$

$3 + -10 = -7$

$(x+3)(x-10)=0$

$x+3=0$ or $x-10=0$

$x=-3$ $\quad x=10$

c) $2y^2 + 11y + 5 = 0$ * Decomposition

$1 \times 10 = 10$

$1 + 10 = 11$

$2y^2 + 1y + 10y + 5 = 0$

$(2y^2 + 1y)(+10y + 5) = 0$

$y(2y + 1) + 5(2y + 1) = 0$

$(2y + 1)(y + 5) = 0$

$2y + 1 = 0$ or $y + 5 = 0$

$\frac{2y}{2} = \frac{-1}{2}$ $\quad y = -5$

$y = \frac{-1}{2}$

d) $4t^2 + 7t - 15 = 0$ * Decomposition

$-5 \times 12 = -60$

$-5 + 12 = 7$

$4t^2 - 5t + 12t - 15 = 0$

$(4t^2 - 5t)(+12t - 15) = 0$

$t(4t - 5) + 3(4t - 5) = 0$

$(4t - 5)(t + 3) = 0$

$4t - 5 = 0$ or $t + 3 = 0$

$\frac{4t}{4} = \frac{5}{4}$ $\quad t = -3$

$t = \frac{5}{4}$

2.

{a-d} a) $x^2 - 121 = 0$ (Difference of Squares)

$(x-11)(x+11)=0$

$x-11=0$ or $x+11=0$

$x=11$ $\quad$ $x=-11$

b) $9r^2 - 100 = 0$ (Difference of Squares)

$(3r-10)(3r+10)=0$

$3r-10=0$ or $3r+10=0$

$\frac{3r}{3} = \frac{10}{3}$ $\quad$ $\frac{3r}{3} = -\frac{10}{3}$

$r = \frac{10}{3}$ $\quad$ $r = -\frac{10}{3}$

c) $x^2 - 15x = 0$ (Common Factor)

$x(x-15) = 0$

$x = 0$ or $x - 15 = 0$

$x = 15$

d) $3y^2 + 48y = 0$ (Common Factor)

$3y(y+16) = 0$

$\frac{3y}{3} = \frac{0}{3}$ or $y + 16 = 0$

$y = 0$ $y = -16$

6. Determine the roots of each equation.

a) $5u^2 - 10u - 315 = 0$

$5(u^2 - 2u - 63) = 0$

$5(u+7)(u-9) = 0$

$u+7=0$ or $u-9=0$

$u=-7$ $\quad u=9$

$7 \times -9 = -63$

$7 + -9 = -2$

b) $0.25x^2 + 1.5x + 2 = 0$

$0.25(x^2 + 6x + 8) = 0$

$0.25(x+4)(x+2) = 0$

$x+4=0$ or $x+2=0$

$x=-4$ $\quad x=-2$

$4 \times 2 = 8$

$4 + 2 = 6$

c) $1.4y^2 + 5.6y - 16.8 = 0$

$1.4(y^2 + 4y - 12) = 0$

$1.4(y+6)(y-2) = 0$

$y+6=0$ or $y-2=0$

$y=-6$ $\quad y=2.$

$6 \times -2 = -12$

$6 + -2 = 4$

d) $\frac{1}{2}K^2 + 5K + 12.5 = 0$

$\frac{1}{2}(K^2 + 10K + 25) = 0$

$\frac{1}{2}(K+5)(K+5) = 0$

$\frac{1}{2}(K+5)^2 = 0$

$K+5=0$

$K=-5$

$5 \times 5 = 25$

$5 + 5 = 10$

7. The graph of a quadratic function has x-intercepts -5 and -12. Write a quadratic equation that has these roots.

$y = a(x-r)(x-s)$

Assuming $a=1$:

$y = (x--5)(x--12)$

$y = (x+5)(x+12)$

$y = x^2+12x+5x+60$

$y = x^2+17x+60$

Quadratic Equation $\Rightarrow x^2+17x+60=0$

10. Identify and correct any errors in the following solution.

$5a^2 - 100 = 0$

$5a^2 = 100$

$a^2 = 25$ ← Error

$\sqrt{a^2} = \sqrt{25}$

$a = 5$ ← Error

Correction:

$5a^2 - 100 = 0$

$\frac{5a^2}{5} = \frac{100}{5}$

$a^2 = 20$

$\sqrt{a^2} = \sqrt{20}$

$a = \pm\sqrt{20}$

11. Identify and correct the errors in this solution:

$4r^2 - 9r = 0$

$(2r-3)(2r+3) = 0$ → Error

$2r - 3 = 0$ or $2r + 3 = 0$

$2r = 3$ $\quad$ $2r = -3$

$r = 1.5$ or $\quad$ $r = -1.5$

Correction:

$4r^2 - 9r = 0$

$r(4r-9) = 0$

$r = 0$ or $4r - 9 = 0$

$\frac{4r}{4} = \frac{9}{4}$

$r = \frac{9}{4}$