$u = x^2$

Ex: $y = \sin(x^2)$

$y' = \cos(x^2) \cdot 2x$

$y' = 2x\cos(x^2)$

$u = \tan x$

① n) $y = \sin(\tan x)$

$y' = \cos(\tan x) \cdot \sec^2 x$

$y' = \sec^2 x\left[\cos(\tan x)\right]$

k) $y = \dfrac{1}{\sqrt{(\sec 2x - 1)^3}} = \dfrac{1}{(\sec 2x - 1)^{3/2}} = (\sec 2x - 1)^{-3/2}$

$y' = -\frac{3}{2}(\sec 2x - 1)^{-5/2} \cdot \sec 2x \tan 2x \cdot (2)$

$y' = \dfrac{-3\sec(2x)\tan(2x)}{(\sec 2x - 1)^{5/2}}$

$y' = -\dfrac{3\sec(2x)\tan(2x)}{\sqrt{(\sec 2x - 1)^5}}$

① a) $y = \tan^2(\cos x) = [\tan(\cos x)]^2$

$y' = 2[\tan(\cos x)] \cdot \sec^2(\cos x) \cdot (-\sin x)(1)$

$y' = -2\sin x\,[\tan(\cos x)][\sec^2(\cos x)]$

b) $y = \dfrac{x^2 \boxed{\tan x}}{\boxed{\sec x}} = x^2\left(\dfrac{\sin x}{\cos x}\right) \cdot \left(\dfrac{\cos x}{1}\right) = x^2 \sin x$

$y' = x^2(\cos x)(1) + 2x(\sin x)$

$y' = x^2\cos x + 2x\sin x$

$y' = x[x\cos x + 2\sin x]$

# Final Review

① b) $f(x) = \boxed{\dfrac{2x-2}{x+3}}$ $\quad f(x+h) = \boxed{\dfrac{2x+2h-2}{x+h+3}}$

$$f'(x) = \lim_{h\to 0} \frac{(x+3)\cancel{(x+h+3)}\dfrac{2x+2h-2}{\cancel{x+h+3}} - \dfrac{2x-2}{\cancel{x+3}}\cancel{(x+3)}(x+h+3)}{h(x+3)(x+h+3)}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2}+\cancel{2xh}-\cancel{2x}+\cancel{6x}+6h-\cancel{6}-(\cancel{2x^2}+\cancel{2xh}+\cancel{6x}-\cancel{2x}-2h-\cancel{6})}{h(x+3)(x+h+3)}$$

$$= \lim_{h\to 0} \frac{8\cancel{h}}{\cancel{h}(x+3)(x+\cancel{h}+3)} = \boxed{\frac{8}{(x+3)^2}}$$

h) $y = (x^2)(\csc x)$

$y' = (x^2)(-\csc x \cot x)(1) + 2x \csc x$

$y' = -x^2 \csc x \cot x + 2x \csc x$

$y' = x \csc x(-x \cot x + 2)$

$y' = x \csc x(2 - x \cot x)$

$u = (1-2x)^2$

$du = 2(1-2x)(-2)$

i) $y = \cot^3(1-2x)^2 = [\cot(1-2x)^2]^3$

$y' = 3[\cot(1-2x)^2]^2[-\csc^2(1-2x)^2][2(1-2x)(-2)]$

$y' = 3\cot^2(1-2x)^2[-\csc^2(1-2x)^2][-4(1-2x)]$

$y' = 12(1-2x)\cot^2(1-2x)^2 \csc^2(1-2x)^2$

k) $y = \dfrac{1}{\sqrt{(\sec 2x - 1)^3}} = \dfrac{1}{(\sec 2x - 1)^{3/2}} = (\sec 2x - 1)^{-3/2}$

$y' = -\dfrac{3}{\cancel{2}}(\sec 2x - 1)^{-5/2}(\sec 2x \tan 2x)(\cancel{2})$

$y' = \dfrac{-3\sec 2x \tan 2x}{(\sec 2x - 1)^{5/2}} = \dfrac{-3\sec 2x \tan 2x}{\sqrt{(\sec 2x - 1)^5}}$

l) $y = \dfrac{x^2 \boxed{\tan x}}{\boxed{\sec x}} = x^2 \cdot \dfrac{\sin x}{\cancel{\cos x}} \cdot \dfrac{\cancel{\cos x}}{1} = x^2 \sin x$

$y' = x^2(\cos x)(1) + 2x\sin x$

$y' = x^2\cos x + 2x\sin x$

$y' = x[x\cos x + 2\sin x]$

$u = \cos x$

$du = -\sin x(1)$

o) $y = \tan^2(\cos x) = [\tan(\cos x)]^2$

$y' = 2[\tan(\cos x)][\sec^2(\cos x)][-\sin x(1)]$

$y' = -2\sin x \tan(\cos x)\sec^2(\cos x)$

⑥ Find the points on the curve $y=\frac{x}{x-1}$ where the tangent line is parallel to the line $x+4y=1$

① $x+4y=1$

$4y=-x+1$

$y=-\frac{1}{4}x+\frac{1}{4}$

$m=-\frac{1}{4}$

② $y=\frac{x}{x-1}$

$x-1 \quad -x$

$y'=\frac{(x-1)(1)-(x)(1)}{(x-1)^2}$

$y'=\frac{-1}{(x-1)^2}$

③ $\frac{-1}{(x-1)^2}=-\frac{1}{4}$

$-(x-1)^2=-4$

$(x-1)^2=4$

$x^2-2x+1=4$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

| | |
|---|---|
| $x-3=0$ | $x+1=0$ |
| $x=3$ | $x=-1$ |

④ $y=\frac{x}{x-1}$

| | |
|---|---|
| $y=\frac{3}{3-1}$ | $y=\frac{-1}{-1-1}$ |
| $y=\frac{3}{2}$ | $y=\frac{1}{2}$ |
| $\boxed{(3, 3/2)}$ | $\boxed{(-1, 1/2)}$ |

⑥ Find the point on the curve $y = x\sqrt{x}$ where the tangent line is parallel to the line $6x - y = 4$

① $6x - y = 4$

$6x - 4 = y$

$y = 6x - 4$

$m = 6$

② $y = x\sqrt{x} = x(x)^{1/2} = x^{3/2}$

$y' = \frac{3}{2}x^{1/2} = \frac{3\sqrt{x}}{2}$

③ $\frac{3\sqrt{x}}{2} = \frac{6}{1}$

$3\sqrt{x} = 12$

$\sqrt{x} = 4$

$x = 16$

④ $y = x\sqrt{x}$

$y = (16)\sqrt{16}$

$y = 64$

$(16, 64)$