Questions from Homework

③ f) $3x-2,\ 3x-5,\ 3x-8,\ \underline{3x-11},\ \underline{3x-14}$

$a = 3x-2$ $\quad t_n = 3x-2+(n-1)(-3)$

$d = -3$ $\quad t_n = 3x-2-3n+3$

$$\boxed{t_n = 3x-3n+1}$$

g) $\frac{2}{x},\ \frac{4}{x},\ \frac{6}{x},\ \frac{8}{x},\ \frac{10}{x}$

$a = \frac{2}{x}$ $\quad t_n = \frac{2}{x}+(n-1)\left(\frac{2}{x}\right)$

$d = \frac{2}{x}$ $\quad t_n = \frac{2}{x}+\frac{2n}{x}-\frac{2}{x}$

$$\boxed{t_n = \frac{2n}{x}}$$

④ b) $a = \frac{1}{2}$ $\quad \frac{1}{2},\ \frac{1}{6},\ -\frac{1}{6}$

$d = -\frac{1}{3}$

$t_2 = \frac{1}{2}+(2-1)\left(-\frac{1}{3}\right)$

$= \frac{1}{2}+(1)\left(-\frac{1}{3}\right)$

$= \frac{1}{2}-\frac{1}{3}$

$= \frac{3}{6}-\frac{2}{6}$

$= \boxed{\frac{1}{6}}$

$t_3 = \frac{1}{2}+(3-1)\left(-\frac{1}{3}\right)$

$= \frac{1}{2}+2\left(\frac{-1}{3}\right)$

$= \frac{1}{2}-\frac{2}{3}$

$= \frac{3}{6}-\frac{4}{6}$

$= \boxed{\frac{-1}{6}}$

Geometric Sequences

Ex: 2, 4, 8, 16, 32

Sequences of numbers that follow a pattern of multiplying a fixed number from one term to the next are called geometric sequences.

- To find the next term, multiply the previous term by a common ratio.
- In the sequence 2, 4, 8, 16, 32 we are multiplying by 2.
- This common ratio is called **"r" ($r = t_2/t_1$).**
- The first term is still called **"a"** or **"t_1".**
- The second term is called **"t_2".**
- The last term or an indicated term is called **"t_n".** (general term)
- The position of a term or the number of terms is called **"n".**

Geometric Sequences

Remember $r = t_2/t_1$

Find "r" and the next term!

1, 2, 4, 8,..., 16

$r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = 2$

16, -8, 4, -2, 1,... $-\frac{1}{2}$

$r = \frac{t_2}{t_1} = \frac{-8}{16} = -\frac{1}{2}$

0.01, 0.06, 0.36, 2.16, ... 12.96 (×6, ×6, ×6, ×6)

$r = \frac{t_2}{t_1} = \frac{0.06}{0.01} = 6$

Geometric Sequences

To find any given term in a geometric sequence we use the following formula:

$$t_n = ar^{n-1}$$

Examples

Find the indicated term

1. 3, 6, 12...

$a = 3 \quad r = 2$

$t_7 = (3)(2)^{7-1}$

$= (3)(2)^6$

$= 3(64)$

$= 192$

2. $2, -1, \frac{1}{2}, \frac{-1}{4} \ldots$

$a = 2 \quad r = -\frac{1}{2}$

$t_9 = (2)\left(\frac{-1}{2}\right)^{9-1}$

$= (2)\left(\frac{-1}{2}\right)^8$

$= 2\left(\frac{1}{256}\right)$

$* \left(\frac{-1}{2}\right)^8 = \frac{(-1)^8}{(2)^8} = \frac{1}{256}$

$= \frac{2}{256}$

$= \frac{1}{128}$

We can also determine the number of terms in the sequence.

$$t_n = ar^{n-1}$$

How many terms are in the following sequences?
(Solve for "n")

9, 27, 81,... 2187 $\quad a=9 \quad r=3 \quad t_n=2187$

$$\frac{2187}{9} = \frac{(9)(3)^{n-1}}{9}$$

$$243 = 3^{n-1}$$

$$3^5 = 3^{n-1}$$

$$5 = n-1$$

$$6 = n$$

$$* \ \frac{\log 243}{\log 3} = 5$$

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots \frac{1}{1024} \quad a=\frac{1}{2} \quad r=\frac{1}{2} \quad t_n=\frac{1}{1024}$

$$\frac{\frac{1}{1024}}{\frac{1}{2}} = \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{n-1}}{\frac{1}{2}}$$

$$\frac{1}{512} = \left(\frac{1}{2}\right)^{n-1}$$

$$\left(\frac{1}{2}\right)^9 = \left(\frac{1}{2}\right)^{n-1}$$

$$9 = n-1$$

$$10 = n$$

$$* \ \frac{\log\left(\frac{1}{512}\right)}{\log\left(\frac{1}{2}\right)} = 9$$

Geometric

Find "a", "r", and "t_n" for the following sequences!

$$t_2 = 12,\ t_5 = 768$$

$t_2 = ar^{2-1}$ $\qquad$ $t_5 = ar^{5-1}$

$t_2 = ar$ $\qquad$ $t_5 = ar^4$

$ar = 12$ $\qquad$ $ar^4 = 768$ $\qquad$ 2x2 system

$$\frac{ar^4 = 768}{ar = 12}$$

$r^3 = 64$

$r = 4$

$ar = 12$

$a(4) = 12$

$4a = 12$

$a = 3$

$t_n = ar^{n-1}$

$t_n = (3)(4)^{n-1}$

$$t_3 = 64,\ t_7 = 4$$

Homework

#1- #6