Differentiation Rules

Product Rule:

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

Express the product rule verbally if you are considering a function of the form...

$f(x)$ = (First) X (Second)

In words, ***the Product Rule*** says that the ***derivative of a product of two functions is:*** ***the first function times the derivative of the second function, plus the derivative of the first function times the second function***

Get in the habit of verbalizing the rule as you differentiate...it will help when the functions get more complicated.

⑥ $f(2)=\underline{\underline{3}}$ $\qquad (fg)'(2) = f(2)g'(2)+f'(2)g(2)$

$f'(2)=\underline{\underline{5}}$ $\qquad = (3)(-4)+(5)(-1)$

$g(2)=\underline{\underline{-1}}$ $\qquad = -12-5$

$g'(2)=\underline{\underline{-4}}$ $\qquad = -17$

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⑤ $y=(2-\sqrt{x})(1+\sqrt{x}+3x)$; $(1,5)$

$= (2-x^{1/2})(1+x^{1/2}+3x)$

(i) $y' = (2-x^{1/2})(\frac{1}{2}x^{-1/2}+3) + (-\frac{1}{2}x^{-1/2})(1+x^{1/2}+3x)$

(ii) $y'(1) = (2-(1)^{1/2})(\frac{1}{2}(1)^{-1/2}+3) + (-\frac{1}{2}(1)^{-1/2})(1+(1)^{1/2}+3(1))$

$y'(1) = (1)(\frac{7}{2}) + (-\frac{1}{2})(5)$

$y'(1) = \frac{7}{2} - \frac{5}{2}$

$y'(1) = 1$ $\longleftarrow$ slope of the tangent "m"

(iii) $y-y_1 = m(x-x_1)$

$y-5 = 1(x-1)$

$y-5 = x-1$

$\boxed{0 = x-y+4}$

Differentiate the following function and simplify your answer:

$$h(t)=(t^3-5t)(6\sqrt{t}-t^{-5})$$

$$= (t^3-5t)(6t^{1/2}-t^{-5})$$

$$h'(t) = (t^3-5t)(3t^{-1/2}+5t^{-6}) + (3t^2-5)(6t^{1/2}-t^{-5})$$

$$= 3t^{5/2}+5t^{-3}-15t^{1/2}-25t^{-5}+18t^{5/2}-3t^{-3}-30t^{1/2}-5t^{-5}$$

$$= 21t^{5/2}-45t^{1/2}+2t^{-3}-20t^{-5}$$

$$\boxed{= 21\sqrt{t^5}-45\sqrt{t}+\frac{2}{t^3}-\frac{20}{t^5}}$$

$$f(x)=(7x^3-x^2+5)(x^9+3x-5)$$

Quotient Rule:

The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

Express the quotient rule verbally if you are considering a function of the form...

$$f(x) = \frac{(\text{First})}{(\text{Second})}$$

In words, ***the Quotient Rule*** says that the ***derivative of a quotient is:***
the denominator times the derivative of the numerator, minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

Examples:

Differentiate the following functions and simplify your answers:

$$F(x)=\frac{x^2+2x-3}{x^3+1}$$

$$F'(x)=\frac{(x^3+1)(2x+2)-(x^2+2x-3)(3x^2)}{(x^3+1)^2}$$

$$=\frac{2x^4+2x^3+2x+2-3x^4-6x^3+9x^2}{(x^3+1)^2}$$

$$=\frac{-x^4-4x^3+9x^2+2x+2}{(x^3+1)^2}$$

$$F(x)=\frac{\sqrt{x}}{1+2x}$$

$$F'(x)=\frac{(1+2x)\left(\frac{1}{2}x^{-1/2}\right)-(x^{1/2})(2)}{(1+2x)^2}$$

$$=\frac{\frac{1}{2}x^{-1/2}+x^{1/2}-2x^{1/2}}{(1+2x)^2}$$

$$=\frac{2\sqrt{x}\,\frac{1}{2\sqrt{x}}-\frac{\sqrt{x}}{1}\,2\sqrt{x}}{2\sqrt{x}\,(1+2x)^2}$$

$$\boxed{=\frac{1-2x}{2\sqrt{x}\,(1+2x)^2}}$$

Differentiate the following functions, do not simplify your answers:

$$f(x)=\frac{8-9x^7}{3x-7}$$

$$f'(x)=\frac{(3x-7)(-63x^6)-(8-9x^7)(3)}{(3x-7)^2}$$

$$f(x)=\frac{x^3-7x^2+2}{x^8-4x^5}$$

$$f'(x)=\frac{(x^8-4x^5)(3x^2-14x)-(x^3-7x^2+2)(8x^7-20x^4)}{(x^8-4x^5)^2}$$

Homework

$$f(x)=\frac{\left(10x^{-5}+x\right)\left(3x^{3}+5\right)}{\left(-2x^{6}+\sqrt[3]{x}\right)}$$

$$f(x)=\frac{(x-7)\left(2x^{6}-x^{4}+5\right)}{\left(6x-x^{5}\right)\left(4x^{3}+2\right)}$$