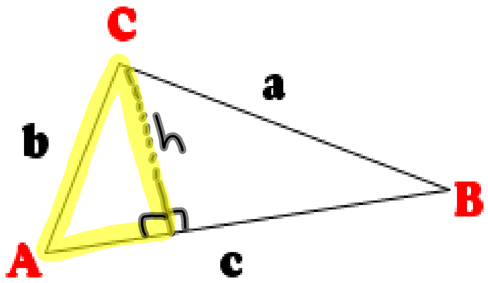
Area of a Triangle

How would you find the area of triangle ABC?



$$\frac{\sin A}{1} = \frac{h}{b}$$

$$h = b\sin A$$

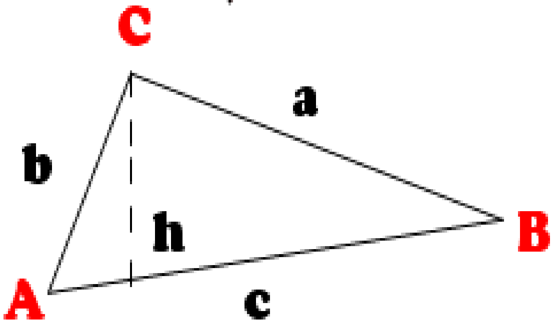
Area = ½ base × height

$$A = \frac{1}{2}(c)(b\sin A)$$

$$\boxed{A = \frac{1}{2}bc\sin A}$$

In this triangle, the base is "c", so

Area = ½ ch



Replace the "h" with an expression using sin A:

$$\sin A = \frac{opp}{hyp}$$

$$\sin A = \frac{h}{b}$$

Solving for h: $h = b \sin A$

If we fill $h = b \sin A$ into our formula, we get:

$$\text{Area} = \tfrac{1}{2}\, c\,(b \sin A)$$

<u>**OR**</u>

$$\text{Area} = \tfrac{1}{2}\, bc \sin A$$

This formula is used to calculate the area of all oblique (non-right) triangles.

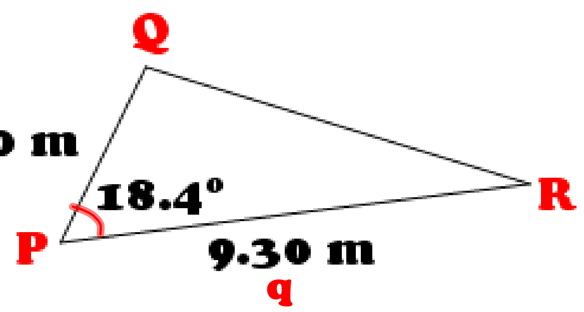
Sometimes finding the area of a right triangle can be done more efficiently using this area formula as well.

To use this formula to find area, you need any 2 sides and the included angle measure of any triangular shape. (You do not need the height!)

When the area of a triangular shape is given, you can use the formula to find any of the missing three measures (b, c, sin A) as long as the other two measures are given.

Example 1:

What is the area of the following triangle?



$$\text{Area} = \frac{1}{2} qr \sin P$$
$$= \frac{1}{2}(9.3)(5.4)(\sin 18.4)$$
$$= \frac{1}{2}(9.3)(5.4)(0.3156)$$
$$= 7.9\,m^2$$

Solution:

A = ½ qr sin P
= ½ (9.30)(5.40)sin 18.4°
= ½ (9.30)(5.40)(0.3156)
= ½ (15.8494)
= 7.92 m² (Watch Units !)

Example 2:

If the area of a triangular region on a stage was to be carpeted with 37 m² of carpet, and two adjacent sides measured 12.0 m and 6.7 m, what is the angle between the two sides.

Area = $37 m^2$ A = ?
b = 12 m
c = 6.7 m

Solution:

$$A = \frac{1}{2} bc \sin A$$

$$37 \text{ m}^2 = \frac{1}{2}(12.0 \text{ m})(6.7 \text{ m})\sin A$$

$$37 \text{ m}^2 = 40.2 \text{ m}^2 \sin A$$

$$\frac{37 \text{ m}^2}{40.2 \text{ m}^2} = \frac{\cancel{40.2 \text{ m}^2} \sin A}{\cancel{40.2 \text{ m}^2}}$$

$$0.9204 = \sin A$$

$$\sin^{-1}(0.9204) = A$$

$$67^\circ = A$$