Sampling Distribution

Sampling Distribution of the Sample Mean

The values of the sample mean are unpredictable and vary from sample to sample. Before you can understand how one sample mean can be used to predict a population mean, you must examine the distribution of all possible values of the sample means. This is called the sampling distribution of the sample mean.

The sampling distribution of the sample mean is the distribution for all possible values of the sample mean that results when random samples of a specific size are repeatedly drawn from a population. This distribution has three unique properties stated in the **Central Limit Theorem**.

Central Limit Theorem

- If samples of size n observations (sample size) are drawn at random from a population with a finite mean and standard deviation, then the sampling distribution of the sample mean $(\bar{x})$ is approximately normal when n is large.

- The mean of this sampling distribution is equal to the population mean: $\mu_{\bar{x}} = \mu$

- The standard deviation of the sample mean is equal to the following: $\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$ pop. s.d.

pop. mean pop s.d.

Example

A sample with a sample size of 30 is taken from a known population where $\mu = 47$ and $\sigma = 4$. The data collected are shown in the following chart.

44.7	47.3	47.2	50.2	52.2	48.4	45.6	51.4	45.3	56.0
46.0	49.4	56.1	41.3	43.1	49.6	44.6	49.7	52.1	44.5
50.1	46.0	43.3	41.0	47.1	52.2	48.0	45.5	48.5	43.4

a) What is the population mean? 47
b) Determine the sample mean.
c) What is the population standard deviation?
d) Determine the sample standard deviation.
e) If you repeatedly collected samples of the same size, what would be the value of the mean of the sample means?
f) If you repeatedly collected samples of the same size, what would be the value of the standard deviation of the sample mean?

Solution

a) The ***population mean*** = 47 (μ) ➡GIVEN!

b) The ***sample mean*** ($\bar{x}$) = $\frac{1429.8}{30}$ = 47.66

c) The ***population standard deviation*** = 4 (σ) ➡GIVEN!

d) (Subtract $\bar{x} = 47.66$)

Data	Difference from Mean	Square of Differences
41.0	-6.66	44.3556
41.3	-6.36	40.4496
43.1	-4.56	20.7936
43.3	-4.36	19.0096
43.4	-4.26	18.1476
44.5	-3.16	9.9856
44.6	-3.06	9.3636
44.7	-2.96	8.7616
45.3	-2.36	5.5696
45.5	-2.16	4.6656
45.6	-2.06	4.2436
46.0	-1.66	2.7556
46.0	-1.66	2.7556
47.1	-0.56	0.3136
47.2	-0.46	0.2116
47.3	-0.36	0.1296
48.0	0.34	0.1156
48.4	0.74	0.5476
48.5	0.84	0.7056
49.4	1.74	3.0276
49.6	1.94	3.7636
49.7	2.04	4.1616
50.1	2.44	5.4516
50.2	2.54	6.4516
51.4	3.74	13.9876
52.1	4.44	19.7136
52.2	4.54	20.6116
52.2	4.54	20.6116
56.0	8.34	69.5556
56.1	8.44	71.2336

Total: 431.9520

Sample Standard Deviation = $\sqrt{\frac{431.9520}{29}}$

= 3.8594 or **3.86**

e) Since you are dealing with the sampling distribution of the sample mean, you know that the mean of the sample means would be equal to the population mean.

$$\mu_{\bar{x}} = \mu = 47$$

f) Again, since you are dealing with the sampling distribution of the sample mean, you know that the standard deviation of the sample mean can be calculated using the formula:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$= \frac{4}{\sqrt{30}}$$

$$= 0.730$$