### Sampling Distribution

### Sampling Distribution of the Sample Mean

The values of the sample mean are unpredictable and vary from sample to sample. Before you can understand how one sample mean can be used to predict a population mean, you must examine the distribution of all possible values of the sample means. This is called the sampling distribution of the sample mean.

The sampling distribution of the sample mean is the distribution for all possible values of the sample mean that results when random samples of a specific size are repeatedly drawn from a population. This distribution has three unique properties stated in the **Central Limit Theorem**.

#### Central Limit Theorem

- If samples of size n observations (sample size) are drawn at random from a population with a finite mean and standard deviation, then the sampling distribution of the sample mean  $(\overline{x})$  is approximately normal when n is large.
- The mean of this sampling distribution is equal to the population mean:  $\mu_{\overline{x}} = \mu$
- The standard deviation of the sample mean is equal to the following:  $\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$

# Example

Popinean pop s.d.

A sample with a sample size of 30 is taken from a known population where  $\mu=47$  and  $\sigma=4$ . The data collected are shown in the following chart.

44.7	47.3	47.2	50.2	52.2	48.4	45.6	51.4	45.3	56.0
46.0	49.4	56.1	41.3	43.1	49.6	44.6	49.7	52.1	44.5
50.1	46.0	43.3	41.0	47.1	52.2	48.0	45.5	48.5	43.4

- a) What is the population mean?
- b) Determine the sample mean.
- c) What is the population standard deviation?
- d) Determine the sample standard deviation.
- e) If you repeatedly collected samples of the same size, what would be the value of the mean of the sample means?
- f) If you repeatedly collected samples of the same size, what would be the value of the standard deviation of the sample mean?

### Solution

- a) The population mean =  $47 (\mu) \Rightarrow GIVEN!$
- b) The sample mean  $(\bar{x}) = \frac{1429.8}{30} = 47.66$
- c) The population standard deviation = 4 (σ) →GIVEN!

## d) (Subtract $\overline{x} = 47.66$ )

41.0   -6.66   44.3556     41.3   -6.36   40.4496     43.1   -4.56   20.7936     43.3   -4.36   19.0096     43.4   -4.26   18.1476     44.5   -3.16   9.9856     44.6   -3.06   9.3636     44.7   -2.96   8.7616     45.3   -2.36   5.5696     45.5   -2.16   4.6656     45.6   -2.06   4.2436     46.0   -1.66   2.7556     47.1   -0.56   0.3136     47.2   -0.46   0.2116     47.3   -0.36   0.1296     48.0   0.34   0.1156     48.4   0.74   0.5476     48.5   0.84   0.7056     49.4   1.74   3.0276     49.6   1.94   3.7636     49.7   2.04   4.1616     50.1   2.44   5.4516     50.2   2.54   6.4516     51.4   3.74   13.9876     52.1   4.44   19.7136 <th>Data</th> <th>Difference from Mean</th> <th colspan="3">Square of Differences</th>	Data	Difference from Mean	Square of Differences		
43.1   -4.56   20.7936     43.3   -4.36   19.0096     43.4   -4.26   18.1476     44.5   -3.16   9.9856     44.6   -3.06   9.3636     44.7   -2.96   8.7616     45.3   -2.36   5.5696     45.5   -2.16   4.6656     45.6   -2.06   4.2436     46.0   -1.66   2.7556     47.1   -0.56   0.3136     47.2   -0.46   0.2116     47.3   -0.36   0.1296     48.0   0.34   0.1156     48.4   0.74   0.5476     48.5   0.84   0.7056     49.4   1.74   3.0276     49.6   1.94   3.7636     49.7   2.04   4.1616     50.1   2.44   5.4516     50.2   2.54   6.4516     51.4   3.74   13.9876     52.1   4.44   19.7136     52.2   4.54   20.6116     56.0   8.34   69.5556			44.3556		
43.3   -4.36   19.0096     43.4   -4.26   18.1476     44.5   -3.16   9.9856     44.6   -3.06   9.3636     44.7   -2.96   8.7616     45.3   -2.36   5.5696     45.5   -2.16   4.6656     45.6   -2.06   4.2436     46.0   -1.66   2.7556     46.0   -1.66   2.7556     47.1   -0.56   0.3136     47.2   -0.46   0.2116     47.3   -0.36   0.1296     48.0   0.34   0.1156     48.4   0.74   0.5476     48.5   0.84   0.7056     49.4   1.74   3.0276     49.6   1.94   3.7636     49.7   2.04   4.1616     50.1   2.44   5.4516     50.2   2.54   6.4516     51.4   3.74   13.9876     52.1   4.44   19.7136     52.2   4.54   20.6116     55.0   8.34   69.5556	41.3	-6.36	40.4496		
43.4   -4.26   18.1476     44.5   -3.16   9.9856     44.6   -3.06   9.3636     44.7   -2.96   8.7616     45.3   -2.36   5.5696     45.5   -2.16   4.6656     45.6   -2.06   4.2436     46.0   -1.66   2.7556     46.0   -1.66   2.7556     47.1   -0.56   0.3136     47.2   -0.46   0.2116     47.3   -0.36   0.1296     48.0   0.34   0.1156     48.4   0.74   0.5476     48.5   0.84   0.7056     49.4   1.74   3.0276     49.6   1.94   3.7636     49.7   2.04   4.1616     50.1   2.44   5.4516     50.2   2.54   6.4516     51.4   3.74   13.9876     52.1   4.54   20.6116     52.2   4.54   20.6116     56.0   8.34   69.5556	43.1	-4.56	20.7936		
44.5   -3.16   9.9856     44.6   -3.06   9.3636     44.7   -2.96   8.7616     45.3   -2.36   5.5696     45.5   -2.16   4.6656     45.6   -2.06   4.2436     46.0   -1.66   2.7556     46.0   -1.66   2.7556     47.1   -0.56   0.3136     47.2   -0.46   0.2116     47.3   -0.36   0.1296     48.0   0.34   0.1156     48.4   0.74   0.5476     48.5   0.84   0.7056     49.4   1.74   3.0276     49.6   1.94   3.7636     49.7   2.04   4.1616     50.1   2.44   5.4516     50.2   2.54   6.4516     51.4   3.74   13.9876     52.1   4.44   19.7136     52.2   4.54   20.6116     56.0   8.34   69.5556	43.3	-4.36	19.0096		
44.6   -3.06   9.3636     44.7   -2.96   8.7616     45.3   -2.36   5.5696     45.5   -2.16   4.6656     45.6   -2.06   4.2436     46.0   -1.66   2.7556     46.0   -1.66   2.7556     47.1   -0.56   0.3136     47.2   -0.46   0.2116     47.3   -0.36   0.1296     48.0   0.34   0.1156     48.4   0.74   0.5476     48.5   0.84   0.7056     49.4   1.74   3.0276     49.6   1.94   3.7636     49.7   2.04   4.1616     50.1   2.44   5.4516     50.2   2.54   6.4516     51.4   3.74   13.9876     52.1   4.44   19.7136     52.2   4.54   20.6116     56.0   8.34   69.5556	43.4	-4.26	18.1476		
44.7   -2.96   8.7616     45.3   -2.36   5.5696     45.5   -2.16   4.6656     45.6   -2.06   4.2436     46.0   -1.66   2.7556     47.1   -0.56   0.3136     47.2   -0.46   0.2116     47.3   -0.36   0.1296     48.0   0.34   0.1156     48.4   0.74   0.5476     48.5   0.84   0.7056     49.4   1.74   3.0276     49.6   1.94   3.7636     49.7   2.04   4.1616     50.1   2.44   5.4516     50.2   2.54   6.4516     51.4   3.74   13.9876     52.1   4.44   19.7136     52.2   4.54   20.6116     56.0   8.34   69.5556	44.5	-3.16	9.9856		
45.3   -2.36   5.5696     45.5   -2.16   4.6656     45.6   -2.06   4.2436     46.0   -1.66   2.7556     46.0   -1.66   2.7556     47.1   -0.56   0.3136     47.2   -0.46   0.2116     47.3   -0.36   0.1296     48.0   0.34   0.1156     48.4   0.74   0.5476     48.5   0.84   0.7056     49.4   1.74   3.0276     49.6   1.94   3.7636     49.7   2.04   4.1616     50.1   2.44   5.4516     50.2   2.54   6.4516     51.4   3.74   13.9876     52.1   4.44   19.7136     52.2   4.54   20.6116     52.2   4.54   20.6116     56.0   8.34   69.5556	44.6	-3.06	9.3636		
45.5   -2.16   4.6656     45.6   -2.06   4.2436     46.0   -1.66   2.7556     46.0   -1.66   2.7556     47.1   -0.56   0.3136     47.2   -0.46   0.2116     47.3   -0.36   0.1296     48.0   0.34   0.1156     48.4   0.74   0.5476     48.5   0.84   0.7056     49.4   1.74   3.0276     49.6   1.94   3.7636     49.7   2.04   4.1616     50.1   2.44   5.4516     50.2   2.54   6.4516     51.4   3.74   13.9876     52.1   4.44   19.7136     52.2   4.54   20.6116     56.0   8.34   69.5556	44.7	-2.96	8.7616		
45.6   -2.06   4.2436     46.0   -1.66   2.7556     46.0   -1.66   2.7556     47.1   -0.56   0.3136     47.2   -0.46   0.2116     47.3   -0.36   0.1296     48.0   0.34   0.1156     48.4   0.74   0.5476     48.5   0.84   0.7056     49.4   1.74   3.0276     49.6   1.94   3.7636     49.7   2.04   4.1616     50.1   2.44   5.4516     50.2   2.54   6.4516     51.4   3.74   13.9876     52.1   4.44   19.7136     52.2   4.54   20.6116     52.2   4.54   20.6116     56.0   8.34   69.5556	45.3	-2.36	5.5696		
46.0   -1.66   2.7556     46.0   -1.66   2.7556     47.1   -0.56   0.3136     47.2   -0.46   0.2116     47.3   -0.36   0.1296     48.0   0.34   0.1156     48.4   0.74   0.5476     48.5   0.84   0.7056     49.4   1.74   3.0276     49.6   1.94   3.7636     49.7   2.04   4.1616     50.1   2.44   5.4516     50.2   2.54   6.4516     51.4   3.74   13.9876     52.1   4.44   19.7136     52.2   4.54   20.6116     52.2   4.54   20.6116     56.0   8.34   69.5556	45.5	-2.16	4.6656		
46.0   -1.66   2.7556     47.1   -0.56   0.3136     47.2   -0.46   0.2116     47.3   -0.36   0.1296     48.0   0.34   0.1156     48.4   0.74   0.5476     48.5   0.84   0.7056     49.4   1.74   3.0276     49.6   1.94   3.7636     49.7   2.04   4.1616     50.1   2.44   5.4516     50.2   2.54   6.4516     51.4   3.74   13.9876     52.1   4.44   19.7136     52.2   4.54   20.6116     52.2   4.54   20.6116     56.0   8.34   69.5556	45.6	-2.06	4.2436		
47.1   -0.56   0.3136     47.2   -0.46   0.2116     47.3   -0.36   0.1296     48.0   0.34   0.1156     48.4   0.74   0.5476     48.5   0.84   0.7056     49.4   1.74   3.0276     49.6   1.94   3.7636     49.7   2.04   4.1616     50.1   2.44   5.4516     50.2   2.54   6.4516     51.4   3.74   13.9876     52.1   4.44   19.7136     52.2   4.54   20.6116     52.2   4.54   20.6116     56.0   8.34   69.5556	46.0	-1.66	2.7556		
47.2   -0.46   0.2116     47.3   -0.36   0.1296     48.0   0.34   0.1156     48.4   0.74   0.5476     48.5   0.84   0.7056     49.4   1.74   3.0276     49.6   1.94   3.7636     49.7   2.04   4.1616     50.1   2.44   5.4516     50.2   2.54   6.4516     51.4   3.74   13.9876     52.1   4.44   19.7136     52.2   4.54   20.6116     52.2   4.54   20.6116     56.0   8.34   69.5556	46.0	-1.66	2.7556		
47.3   -0.36   0.1296     48.0   0.34   0.1156     48.4   0.74   0.5476     48.5   0.84   0.7056     49.4   1.74   3.0276     49.6   1.94   3.7636     49.7   2.04   4.1616     50.1   2.44   5.4516     50.2   2.54   6.4516     51.4   3.74   13.9876     52.1   4.44   19.7136     52.2   4.54   20.6116     52.2   4.54   20.6116     56.0   8.34   69.5556	47.1	-0.56	0.3136		
48.0   0.34   0.1156     48.4   0.74   0.5476     48.5   0.84   0.7056     49.4   1.74   3.0276     49.6   1.94   3.7636     49.7   2.04   4.1616     50.1   2.44   5.4516     50.2   2.54   6.4516     51.4   3.74   13.9876     52.1   4.44   19.7136     52.2   4.54   20.6116     52.2   4.54   20.6116     56.0   8.34   69.5556	47.2	-0.46	0.2116		
48.4   0.74   0.5476     48.5   0.84   0.7056     49.4   1.74   3.0276     49.6   1.94   3.7636     49.7   2.04   4.1616     50.1   2.44   5.4516     50.2   2.54   6.4516     51.4   3.74   13.9876     52.1   4.44   19.7136     52.2   4.54   20.6116     52.2   4.54   20.6116     56.0   8.34   69.5556	47.3	-0.36	0.1296		
48.5 0.84 0.7056   49.4 1.74 3.0276   49.6 1.94 3.7636   49.7 2.04 4.1616   50.1 2.44 5.4516   50.2 2.54 6.4516   51.4 3.74 13.9876   52.1 4.44 19.7136   52.2 4.54 20.6116   52.2 4.54 20.6116   56.0 8.34 69.5556	48.0	0.34	0.1156		
49.4 1.74 3.0276   49.6 1.94 3.7636   49.7 2.04 4.1616   50.1 2.44 5.4516   50.2 2.54 6.4516   51.4 3.74 13.9876   52.1 4.44 19.7136   52.2 4.54 20.6116   52.2 4.54 20.6116   56.0 8.34 69.5556	48.4	0.74			
49.6 1.94 3.7636   49.7 2.04 4.1616   50.1 2.44 5.4516   50.2 2.54 6.4516   51.4 3.74 13.9876   52.1 4.44 19.7136   52.2 4.54 20.6116   52.2 4.54 20.6116   56.0 8.34 69.5556	48.5	0.84	0.7056		
49.7 2.04 4.1616   50.1 2.44 5.4516   50.2 2.54 6.4516   51.4 3.74 13.9876   52.1 4.44 19.7136   52.2 4.54 20.6116   52.2 4.54 20.6116   56.0 8.34 69.5556	49.4	1.74	3.0276		
50.1 2.44 5.4516   50.2 2.54 6.4516   51.4 3.74 13.9876   52.1 4.44 19.7136   52.2 4.54 20.6116   52.2 4.54 20.6116   56.0 8.34 69.5556	49.6	1.94	3.7636		
50.2 2.54 6.4516   51.4 3.74 13.9876   52.1 4.44 19.7136   52.2 4.54 20.6116   52.2 4.54 20.6116   56.0 8.34 69.5556	49.7	2.04	4.1616		
51.4 3.74 13.9876   52.1 4.44 19.7136   52.2 4.54 20.6116   52.2 4.54 20.6116   56.0 8.34 69.5556	50.1	2.44	5.4516		
52.1 4.44 19.7136   52.2 4.54 20.6116   52.2 4.54 20.6116   56.0 8.34 69.5556	50.2	2.54	6.4516		
52.2 4.54 20.6116   52.2 4.54 20.6116   56.0 8.34 69.5556	51.4	3.74	13.9876		
52.2 4.54 20.6116   56.0 8.34 69.5556		4.44	19.7136		
56.0 8.34 69.5556	52.2	4.54	20.6116		
	52.2	4.54	20.6116		
56.1 8.44 71.2336	56.0	8.34	69.5556		
	56.1	8.44	71.2336		

Total: 431.9520

Sample Standard Deviation = 
$$\sqrt{\frac{431.9520}{29}}$$
  
= 3.8594 or **3.86**

e) Since you are dealing with the sampling distribution of the sample mean, you know that the mean of the sample means would be equal to the population mean.

$$\mu_{\overline{x}} = \mu = 47$$

f) Again, since you are dealing with the sampling distribution of the sample mean, you know that the standard deviation of the sample mean can be calculated using the formula:

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

$$=\frac{4}{\sqrt{30}}$$

$$= 0.730$$