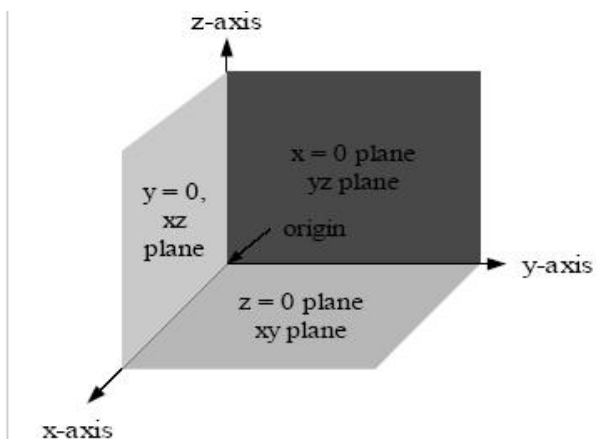


# ALGEBRA OF 3-SPACE

- Coordinate geometry that represents space in **three** dimensions
- Coordinates are in the form of an ordered triplet (**x, y, z**)
- Three planes exist: **xy** plane, **xz** plane, **yz** plane

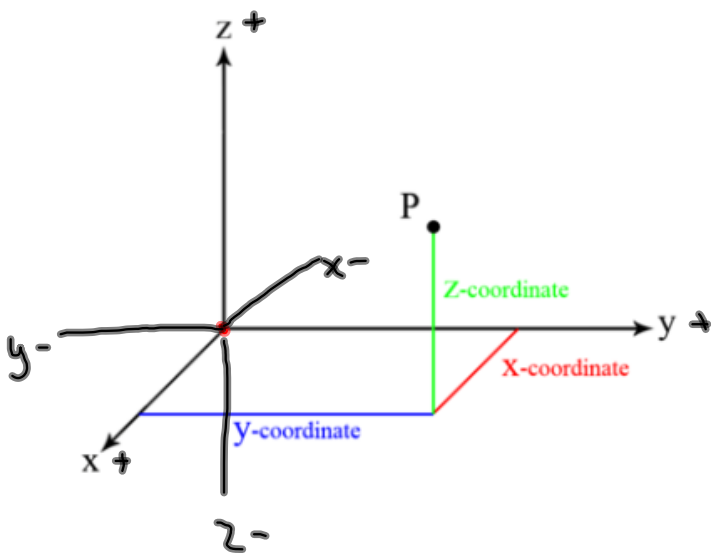


x axis -axis coming "out of the page"

y axis - horizontal axis

z axis - vertical axis

## Plotting Points in 3-Space



$(2, 3, 2)$

## Solving 3 x 3 Systems

### **REMEMBER:**

- you can multiply equations by a constant
- can add & subtract 2 equations to get a new equation
- you can rearrange the order of equations

### **STEPS:**

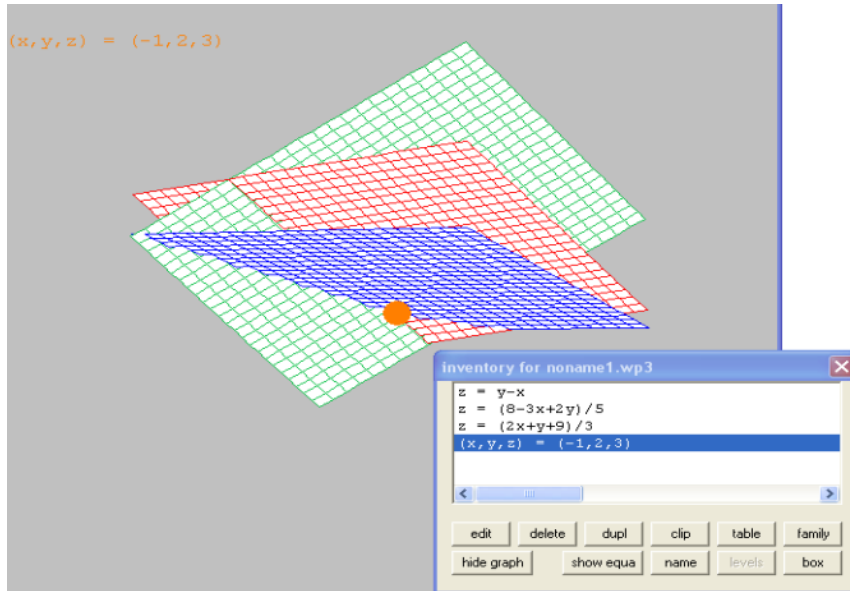
- 1) Eliminate one of the variables
- 2) Solve the 2 x 2 system
- 3) Use "backward substitution" to obtain a solution

# Types of Systems

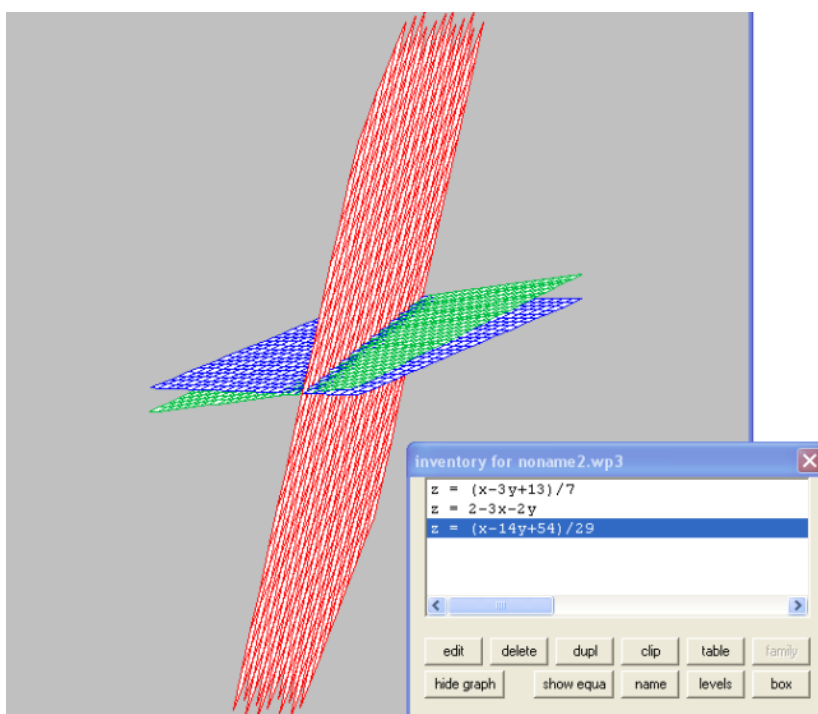
Remember: Looking at **intersecting planes!**

**Consistent:**

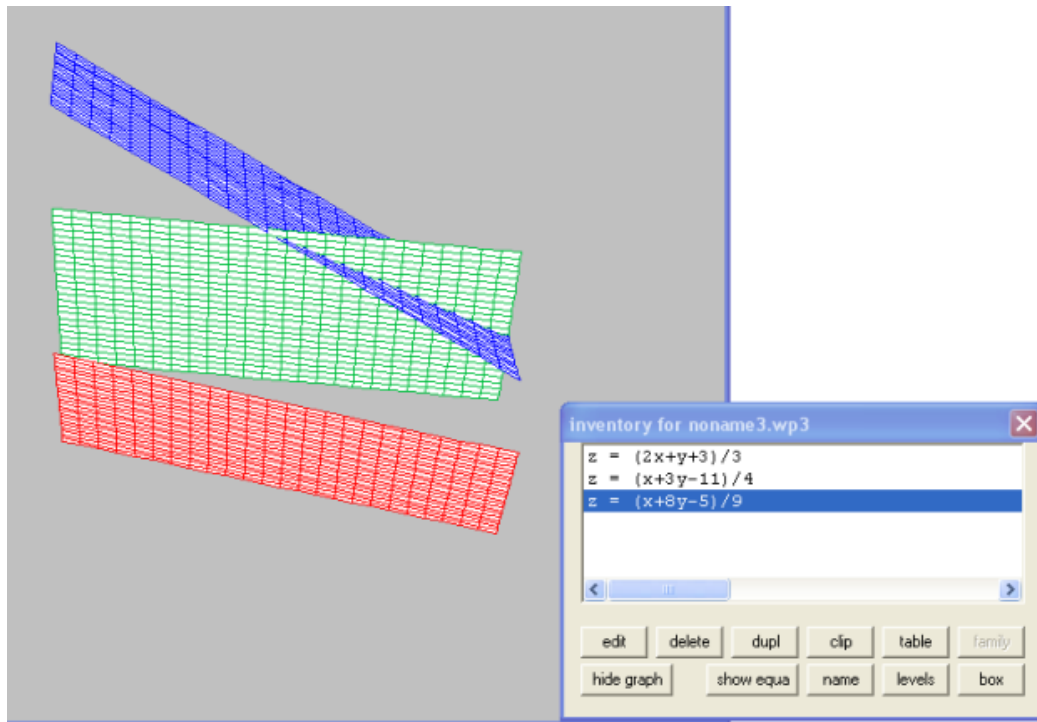
Independent: one unique solution



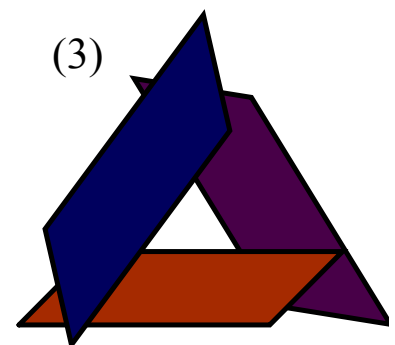
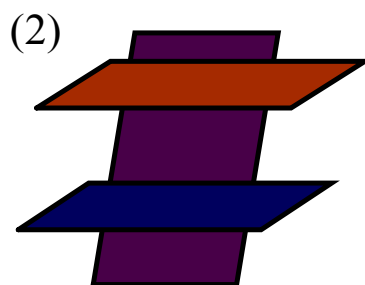
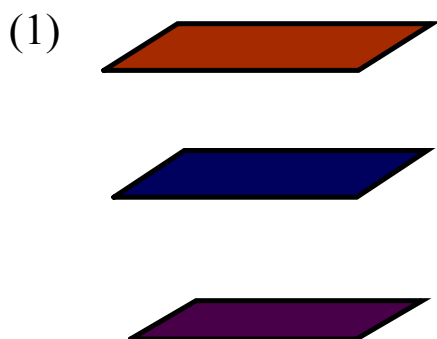
Dependent: Infinite number of solutions



## Inconsistent: No Solutions



3 Possible Orientations That Give No Solution...



## I. Consistent System with a Unique Solution

Solve using algebraic techniques

$$3x - 2y = 6$$

$$5x - 9y + 5z = -36$$

$$x - 6y + 7z = -39$$

$$35x - 63y + 35z = -252$$

$$\Leftrightarrow 5x - 30y + 35z = -195$$

$$30x - 33y = -57$$

$$\begin{array}{r} 30x - 33y = -57 \\ \Leftrightarrow 30x - 20y = 60 \\ \hline -13y = -117 \\ y = 9 \end{array}$$

$$\begin{array}{r} 3x - 2y = 6 \\ 3x - 2(9) = 6 \\ 3x - 18 = 6 \\ 3x = 24 \\ x = 8 \end{array}$$

$$\begin{array}{r} x - 6y + 7z = -39 \\ 8 - 6(9) + 7z = -39 \\ 8 - 54 + 7z = -39 \\ -46 + 7z = -39 \\ 7z = 7 \\ z = 1 \end{array}$$

$$(8, 9, 1)$$

## I. Consistent System with a Unique Solution

Solve the following system of equations using a matrix reduced to its row echelon form.

$$4x + 3y - z = -7$$

$$3x - 2y + 3z = -10$$

$$x + y - z = -2$$

$$\left[ \begin{array}{ccc|c} 4 & 3 & -1 & -7 \\ 3 & -2 & 3 & -10 \\ 1 & 1 & -1 & -2 \end{array} \right] \xrightarrow{\substack{R_2 - 3R_3 \\ 4R_3 - R_1}} \left[ \begin{array}{ccc|c} 4 & 3 & -1 & -7 \\ 0 & -5 & 6 & -4 \\ 0 & 1 & -3 & -1 \end{array} \right] \xrightarrow{5R_3 + R_2} \left[ \begin{array}{ccc|c} 4 & 3 & -1 & -7 \\ 0 & -5 & 6 & -4 \\ 0 & 0 & -9 & -9 \end{array} \right]$$

$$-9z = -9$$

$$\boxed{z = 1}$$

$$-5y + 6z = -4$$

$$-5y + 6(1) = -4$$

$$-5y = -10$$

$$\boxed{y = 2}$$

$$4x + 3y - z = -7$$

$$4x + 3(2) - 1 = -7$$


$$4x + 6 - 1 = -7$$

$$4x = -12$$

$$\boxed{x = -3}$$

$$\boxed{(-3, 2, 1)}$$

## Word Problems

The San Diego Chargers football team uses three brands of cleats each year: Nike, Adidas, and Reebok. Last year the team went through a total of 410 pairs of cleats. Nike's cost \$84/pair, Adidas \$72/pair and Reeboks \$65/pair and they spent \$31 050 on cleats last season. If Nike's cleats were used twice as much as Reeboks, how many pairs of each brand of football cleat did they use? 

(Declare variables, write a system of equations and an augmented matrix to model the problem then use your TI-84 to solve.)

Let  $x = \text{Nike}$

Let  $y = \text{Adidas}$

Let  $z = \text{Reebok}$

$$x = 2z$$

$$x - 2z = 0$$

$$84x + 72y + 65z = 31050$$

$$x + y + z = 410$$

$$x - 2z = 0$$

$$\left[ \begin{array}{ccc|c} 84 & 72 & 65 & 31050 \\ 1 & 1 & 1 & 410 \\ 1 & 0 & -2 & 0 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 180 \\ 0 & 1 & 0 & 140 \\ 0 & 0 & 1 & 90 \end{array} \right]$$

$\therefore$  They used 180 pairs of Nike cleats, 140 pairs of Adidas and 90 pairs of Reebok.



## II. Consistent System with a Dependent Solution (must create a parametric solution)

$$\begin{array}{l}
 \textcircled{1} \quad x - 3y - 7z = -13 \\
 \textcircled{2} \quad 3x + 2y + z = 2 \\
 \textcircled{3} \quad x - 14y - 29z = -54
 \end{array}
 \quad
 \begin{array}{l}
 3x - 9y - 21z = -39 \\
 \Leftrightarrow \frac{3x + 2y + z = 2}{-11y - 22z = -41} \\
 \textcircled{4} \quad -11y - 22z = -41
 \end{array}
 \quad
 \begin{array}{l}
 3x + 2y + z = 2 \\
 \Leftrightarrow \frac{3x - 42y - 81z = 162}{44y + 88z = 164} \\
 \textcircled{5} \quad 44y + 88z = 164
 \end{array}$$

$$\begin{array}{l}
 \textcircled{4} \quad -44y - 88z = -164 \\
 \textcircled{5} \quad \frac{44y + 88z = 164}{0 = 0}
 \end{array}
 \quad
 \boxed{\text{let } z = t}$$

$$\begin{array}{l}
 -11y - 22t = -41 \\
 -11y = -41 + 22t \\
 \boxed{y = \frac{41 - 22t}{11}}
 \end{array}$$

$$x - 3y - 7z = -13$$

$$x - 3\left(\frac{41 - 22t}{11}\right) - 7t = -13$$

$$x - \frac{123 + 66t}{11} - 7t = -13$$

$$11x - 123 + 66t - 77t = -143$$

$$11x = -20 + 11t$$

$$\boxed{x = \frac{-20 + 11t}{11}}$$

Don't forget about **Matrices**:

- Basic operations
- Determinants
- Identity Matrix
- Inverse Matrices
- Operations with TI-83
- Row Reduced Echelon Form

$$\textcircled{1} \begin{bmatrix} \underline{2x+y} & 5 \\ -1 & \underline{y-x} \end{bmatrix} = \begin{bmatrix} \underline{-2} & 5 \\ -1 & \underline{1} \end{bmatrix}$$

$$\begin{array}{l} 2x+y = -2 \\ \leftrightarrow -x+y = 1 \\ \hline 3x = -3 \\ \boxed{x = -1} \end{array} \quad \begin{array}{l} -x+y = 1 \\ -(-1)+y = 1 \\ 1+y = 1 \\ \boxed{y = 0} \end{array}$$

⑦ Determinant Method:

$$\begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$$

① Det =  $ad - bc$   
 $= (7)(1) - (3)(2)$   
 $= 7 - 6$   
 $= 1$

② New Matrix  $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$   
 $\begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$

③ Inverse:  
 $\frac{1}{\det} \times [\text{New Matrix}]$   
 $\frac{1}{1} \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$   
 $\begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$

⑧ Let  $x =$  amount invested @ 8%  
 Let  $y =$  " " @ 10%  
 Let  $z =$  " " @ 16%

$$\begin{aligned} x + y + z &= 9000 \\ 0.08x + 0.1y + 0.16z &= 1160 \\ -0.08x - 0.1y + 0.16z &= 440 \end{aligned}$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 9000 \\ 0.08 & 0.1 & 0.16 & 1160 \\ -0.08 & -0.1 & 0.16 & 440 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 2000 \\ 0 & 1 & 0 & 2000 \\ 0 & 0 & 1 & 5000 \end{array} \right]$$