Point Estimates and Confidence Intervals

The objective of inferential statistics is to use sample data to increase knowledge about the corresponding population. Often we attempt to use a sample mean obtained from a single sample to represent the population mean. A single sample mean is called a ***point estimate*** because this single number is used as a plausible value of the population mean.

Suppose that, instead of reporting a point estimate (sample mean) as the single most credible value of the population mean, you report an interval of reasonable values based on the sample data. This ***interval estimator*** of the population mean is called the ***confidence interval***.

EXAMPLE

Colin randomly selected 60 cookies from one product line and had those cookies analyzed for the number of grams of fat they each contained. Rather than reporting the sample mean (point estimate), he reported the confidence interval (interval estimator). Colin reported that the population mean fat content in one cookie is between 10.3 g and 12.1 g with 95% confidence.

TRANSLATION

The population mean refers to the unknown population mean. The confidence interval is from 10.3 to 12.1 and Colin is using a 95% confidence level. This means that the method that produced this interval from 10.3 to 12.1 has a 0.95 probability of enclosing the population mean. ***It does not mean that there is a 0.95 probability that the population mean falls in the interval from 10.3 to 12.1***. This point is important because you must remember that the population mean is fixed and the sample is random.

CONFIDENCE INTERVAL CALCULATION

The confidence interval for the population mean can be calculated using the following formula: $\bar{x} \pm z\frac{\boldsymbol{\sigma}}{\sqrt{\boldsymbol{n}}}$

where z is
- 1.645 for a 90% confidence interval
- 1.96 for a 95% confidence interval
- 2.56 for a 99% confidence interval

However, to use this formula, you must know the population standard deviation (**σ**) in order to calculate the interval. Unfortunately, you rarely know **σ**, so there must be another approach. If the sample size is large, then you can replace the population standard deviation with the sample standard deviation so that you obtain the following formula: $\bar{x} \pm z\frac{\boldsymbol{S_x}}{\sqrt{\boldsymbol{n}}}$

EXAMPLE **(σ known)**

Zach collects four samples of size 60 from a know population with a population mean of 110 and a population standard deviation of 19. He calculates the four sample means based on each of the four samples and records the results in the chart below.

107	115	109	112

a) Determine a 90% confidence interval for each of these samples.
b) Do all four confidence intervals enclose the population mean?

SOLUTION

a)

$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$

$107 \pm 1.645\frac{19}{\sqrt{60}}$

107 ± 4.04

from 102.96 to 111.04

$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$

$115 \pm 1.645\frac{19}{\sqrt{60}}$

115 ± 4.04

from 110.96 to 119.04

$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$

$109 \pm 1.645\frac{19}{\sqrt{60}}$

109 ± 4.04

from 104.96 to 113.04

$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$

$112 \pm 1.645\frac{19}{\sqrt{60}}$

112 ± 4.04

from 107.96 to 116.04

b) Three of the four 90% confidence intervals enclose the population mean.
The interval from 110.96 to 119.04 does not enclose the population mean of 110.

<u>EXAMPLE</u> (σ unknown)

Tim collected a random sample of size 80 and found that the sample mean is 127 and the sample standard deviation is 15. Determine the 95% confidence interval.

<u>SOLUTION</u>

$$\bar{x} \pm z\frac{\boldsymbol{S_x}}{\sqrt{\boldsymbol{n}}}$$

$$127 \pm 1.96\,\frac{\mathbf{15}}{\sqrt{\mathbf{80}}}$$

$$127 \pm 3.29$$

The 95% confidence interval is between 123.71 and 130.29.