

# Related Rates Review #1

① a)  $x^2 + 2xy + y^2 = 7$

$$2x + 2xy' + 2y + 2yy' = 0$$

$$2xy' + 2yy' = -2x - 2y$$

$$y'(2x + 2y) = -2(x + y)$$

$$y' = \frac{-2(x+y)}{2(x+y)}$$

$$y' = -1$$

b)  $6x^3 + 3x^2y = 4y^3$

$$18x^2 + 3x^2y' + 6xy' = 8yy'$$

$$18x^2 + 6xy' = 8yy' - 3x^2y'$$

$$18x^2 + 6xy' = y'(8y - 3x^2)$$

$$\frac{18x^2 + 6xy}{8y - 3x^2} = y'$$

②  $s = t^3 - 4.5t^2 + 6t$

$$v = 3t^2 - 9t + 6$$

$$a = 6t - 9$$

a)  $v(3) = 3(3)^2 - 9(3) + 6$

$$= 27 - 27 + 6$$

$$= 6 \text{ m/s}$$

b)  $a(4) = 6(4) - 9$

$$= 24 - 9$$

$$= 15 \text{ m/s}^2$$

c)  $0 = 3t^2 - 9t + 6$

$$0 = 3(t^2 - 3t + 2)$$

$$0 = 3(t-1)(t-2)$$

$$t = 1 \text{ s and } 2 \text{ s}$$

d)  $6t - 9 > 0$

$$6t > 9$$

$$t > 1.5 \text{ s}$$

③  $\frac{dV}{dt} = 2 \text{ m}^3/\text{min}$

$$h = 4$$

$$r = 2h$$

$$\frac{dh}{dt} = ?$$

$$V = \frac{1}{3}\pi r^2 h \rightarrow \frac{dV}{dt} = 4\pi h^2 \frac{dh}{dt}$$

$$V = \frac{1}{3}\pi (2h)^2 h \rightarrow 2 = 4\pi (4)^2 \frac{dh}{dt}$$

$$V = \frac{4}{3}\pi h^3$$

$$\frac{1 \text{ m}^3/\text{min}}{32\pi} = \frac{dh}{dt}$$

④  $l = 5 \text{ cm}$

$$\frac{dV}{dt} = 125 \text{ cm}^3/\text{s}$$

$$V = l^3$$

$$\frac{dV}{dt} = 3l^2 \frac{dl}{dt}$$

$$125 = 3(5)^2 \frac{dl}{dt}$$

$$1.7 \text{ cm/s} = \frac{dl}{dt}$$

⑤  $\frac{dr}{dt} = 10 \text{ cm/s}$

$$r = 10 \times 3 = 30 \text{ cm}$$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(30)(10)$$

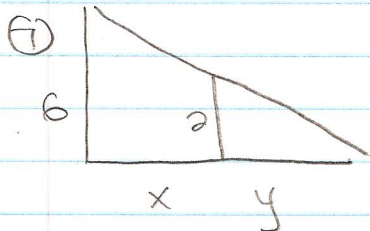
$$\frac{dA}{dt} = 600\pi \text{ cm}^2/\text{s}$$

# Related Rates Review #1

⑥  $\frac{dV}{dt} = 6 \text{ m}^3/\text{min}$       $V = \frac{4}{3}\pi r^3$       $6 = 4\pi(r)^2 \frac{dr}{dt}$

$r = 2 \text{ m}$       $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$       $0.119 \text{ m/min} = \frac{dr}{dt}$

$\frac{dr}{dt} = ?$

⑦   $\frac{dx}{dt} = -1.5 \text{ m/s}$       $\frac{6}{x+y} = \frac{a}{y}$       $2(-1.5) = 4 \frac{dy}{dt}$

$\frac{dy}{dt} = ?$       $2x + 2y = 6y$       $-0.75 \text{ m/s} = \frac{dy}{dt}$

$2x = 4y$       $\frac{2dx}{dt} = 4 \frac{dy}{dt}$

⑧  $\frac{dx}{dt} = ?$      Solve for x:      $x^2 + y^2 = 22$

$\frac{dy}{dt} = 4$       $x^2(2) + (2)^2 = 22$       $x^2 y + y^2 = 22$

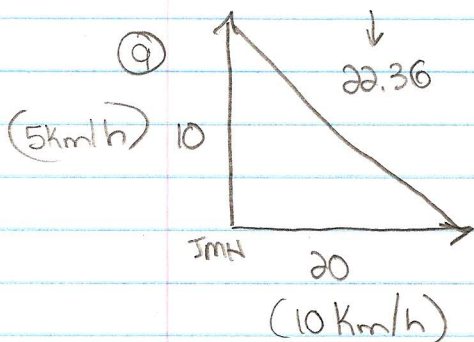
$y = 2$       $2x^2 = 18$       $x^2 \frac{dy}{dt} + 2x \frac{dx}{dt} y + 2y \frac{dy}{dt} = 0$

$x = 3$       $(9)(4) + 2(3)(\frac{dx}{dt})(2) + 2(2)(4) = 0$

$12 \frac{dx}{dt} = -52$

$\frac{dx}{dt} = -4.3$

use pythagorean



$x^2 + y^2 = z^2$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$

$2(20)(10) + 2(10)(5) + 2(22.36) \frac{dz}{dt}$

$400 + 100 = 44.72 \frac{dz}{dt}$

$11.18 \text{ km/h} = \frac{dz}{dt}$

# Related Rates Review #2

① a)  $x^3 + 2x^2y + y^3 = 12$   
 $3x^2 + 2x^2y' + 4xy + 3y^2y' = 0$   
 $2x^2y' + 3y^2y' = -3x^2 - 4xy$   
 $y'(2x^2 + 3y^2) = -3x^2 - 4xy$   
 $y' = \frac{-3x^2 - 4xy}{2x^2 + 3y^2}$

b)  $2xy^2 - y^3 = x^2$   
 $4xyy' + 2y^2 - 3y^2y' = 2x$   
 $4xyy' - 3y^2y' = 2x - 2y^2$   
 $y'(4xy - 3y^2) = 2x - 2y^2$   
 $y' = \frac{2x - 2y^2}{4xy - 3y^2}$

②  $s = 2t^3 - 2t^2 + 60t$   
 $v = 6t^2 - 4t + 60$   
 $a = 12t - 4$

a)  $v(3) = 6(3)^2 - 4(3) + 60$   
 $= 54 - 12 + 60$   
 $= 102 \text{ m/s}$

b)  $0 = 6t^2 - 4t + 60$   
 $0 = 6(t^2 - 7t + 10)$   
 $0 = 6(t-2)(t-5)$   
 $t = 2 \text{ s and } 5 \text{ s}$

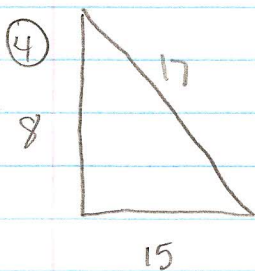
c)  $a(4) = 12(4) - 4$   
 $= 48 - 4$   
 $= 44 \text{ m/s}^2$

d)  $12t - 4 > 0$   
 $12t > 4$   
 $t > 3.5 \text{ s}$

③  $\frac{dA}{dt} = -6 \text{ m}^2/\text{hour}$   
 $r = 3 \text{ m}$   
 $\frac{dr}{dt} = ?$

$A = 4\pi r^2$   
 $\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$   
 $-6 = 8\pi(3) \frac{dr}{dt}$

$\frac{-1}{4\pi} = \frac{dr}{dt} = -0.0796 \text{ m/h}$



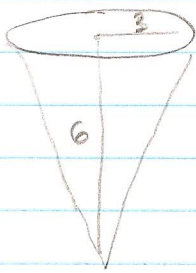
$\frac{dy}{dt} = -4 \text{ m/s}$   
 $\frac{dx}{dt} = ?$

$x^2 + y^2 = (17)^2$   
 $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$   
 $2(15) \frac{dx}{dt} + 2(8)(-4) = 0$

$30 \frac{dx}{dt} = 64$   
 $\frac{dx}{dt} = 2.13 \text{ m/s}$

# Related Rates Review #2

5



$$\frac{r}{h} = \frac{3}{6}$$

$$r = \frac{3}{6}h$$

$$r = \frac{1}{2}h$$

$$\frac{dV}{dt} = \pi \text{ m}^3/\text{min}$$

$$h = 3\text{m}$$

$$\frac{dh}{dt} = ?$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{1}{2}h\right)^2 h$$

$$V = \frac{1}{12} \pi h^3$$

$$\frac{dV}{dt} = \frac{1}{4} \pi h^2 \frac{dh}{dt}$$

$$\pi = \frac{1}{4} \pi (3)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = 0.4 \text{ m/min}$$

6

$$\frac{dV}{dt} = 12 \text{ m}^3/\text{min}$$

$$r = 4\text{m}$$

$$\frac{dr}{dt} = ?$$

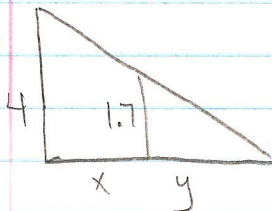
$$V = \frac{4}{3} \pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$12 = 4\pi (4)^2 \frac{dr}{dt}$$

$$0.0596 \text{ m/min} = \frac{dr}{dt}$$

7



$$\frac{dx}{dt} = -0.8 \text{ m/s}$$

$$\frac{dy}{dt} = ?$$

$$\frac{4}{x+y} = \frac{1.7}{y}$$

$$1.7x + 1.7y = 4y$$

$$1.7x = 2.3y$$

$$1.7 \frac{dx}{dt} = 2.3 \frac{dy}{dt}$$

$$1.7(-0.8) = 2.3 \frac{dy}{dt}$$

$$-0.59 \text{ m/s} = \frac{dy}{dt}$$

8

$$x = y^3 + y$$

$$1 = 3y^2 y' + y'$$

$$1 = y'(3y^2 + 1)$$

$$y' = \frac{1}{(3y^2 + 1)} = (3y^2 + 1)^{-1}$$

$$y'' = -(3y^2 + 1)^{-2} (6yy')$$

$$y'' = \frac{-6yy'}{(3y^2 + 1)^2}$$

$$y'' = \frac{-6y(3y^2 + 1)^{-1}}{(3y^2 + 1)^2}$$

$$= \frac{-6y}{(3y^2 + 1)^3}$$