SOLUTIONS ⇒ CHAPTERS 6-8 CUMULATIVE TEST (1-4, 6, 9-19, 21, 22, 24, 26, 28-30)

MULTIPLE CHOICE

1. Consider this linear inequality: $3x + 2y \geq 5$, $x \in I$, $y \in I$

What type of boundary line is used for the graph of the solution set?

A. solid **B.** dashed **C.** stippled **D.** dashed and stippled

2. A system of linear inequalities is defined by

$\{(x, y) \mid 3x - 2y < 12, x \in R, y \in R\}$

$\{(x, y) \mid x + y \geq 5, x \in R, y \in R\}$

Which of these points is in the solution set of the system?

A. (5, 2.5) **B.** (5.5, −1) **C.** (6, 3) **D.** (1, 2)

A.

L.S	R.S
$3x-2y$	12
$3(5)-2(2.5)$	
$15-5$	
10 ✓	

L.S	R.S
$x+y$	5
$(5)+(2.5)$	
7.5 ✓	

B.

L.S	R.S
$3x-2y$	12
$3(5.5)-2(-1)$	
$16.5+2$	
18.5 ✗	

L.S	R.S
$x+y$	5
$(5.5)+(-1)$	
4.5 ✗	

C.

L.S	R.S
$3x-2y$	12
$3(6)-2(3)$	
$18-6$	
12 ✗	

L.S	R.S
$x+y$	5
$6+3$	
9 ✗	

D.

L.S	R.S
$3x-2y$	12
$3(1)-2(2)$	
$3-4$	
-1 ✓	

L.S	R.S
$x+y$	5
$1+2$	
3 ✗	

3. Which graph represents the solution set of the given system?

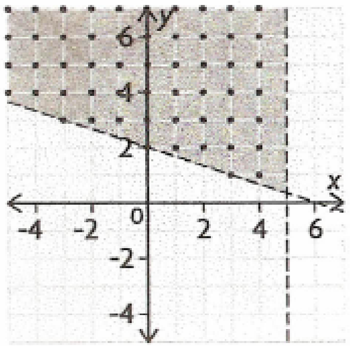
$\{(x, y) \mid 3y + x \geq 6, x \in \mathbb{W}, y \in \mathbb{W}\}$
$\{(x, y) \mid x < 5, x \in \mathbb{W}, y \in \mathbb{W}\}$

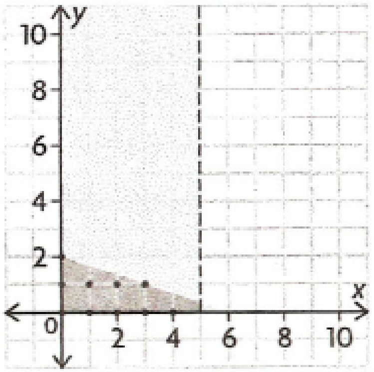
Quadrant I

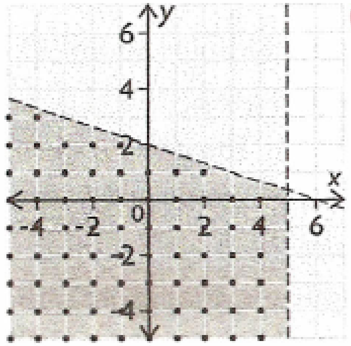
Test Point; (0,0)

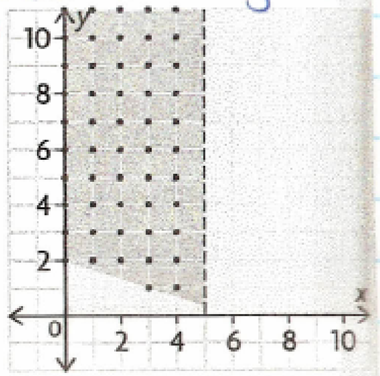
L.S	R.S
3(0)+0	6
0	

0 is not ≥6 ⇒ (0,0) not included in solution region.

A. 

B. 

C. 

D. 

4. The quadratic function $f(x) = 2(x - 4)^2 - 8$ is in which form?

A. factored form

B. vertex form

C. partially factored form

D. standard form

5. What is the vertex of the parabola defined by $y = (3 - x)(x + 2)$?

A. (1, 3) B. (−1, −4) C. (−0.5, −6.25) D. (0.5, 6.25)

$$y = (3-x)(x+2)$$
$$\Rightarrow y = (-x+3)(x+2)$$
$$y = -(x-3)(x+2)$$
x-ints => (3,0) & (-2,0)

x-coordinate of vertex: $\frac{3-2}{2}$
$= \frac{1}{2}$
$= 0.5$

$$y = (3-x)(x+2)$$
$$y = (3-0.5)(0.5+2)$$
$$y = (2.5)(2.5)$$
$$y = 6.25$$

* Vertex (0.5, 6.25)

6. Which of these quadratic equations has one real root?

A. $2x^2 + 6x - 5 = 0$

B. $3x^2 = 30x - 75$

C. $x(x - 28) = 196$

D. $4(x + 2)^2 - 9 = 0$

A. $2x^2+6x-5=0$

$a=2, b=6, c=-5$

$\hookrightarrow b^2-4ac$

$(6)^2-4(2)(-5)$

$=36+40$

$=72$

B. $3x^2=30x-75$

$3x^2-30x+75=0$

$a=3, b=-30, c=75$

b^2-4ac

$=(-30)^2-4(3)(75)$

$=900-900$

$=0$

C. $x(x-28)=196$

$x^2-28x-196=0$

$a=1, b=-28, c=-196$

$b^2-4ac=0$

$=(-28)^2-4(1)(-196)$

$=784+784$

$=1568$

D. $4(x+2)^2-9=0$

$4(x+2)(x+2)-9=0$

$(4x+8)(x+2)-9=0$

$4x^2+8x+8x+16-9=0$

$4x^2+16x+7=0$

$\to b^2-4ac$

$=(16)^2-4(4)(7)$

$=256-112$

$=144$

9. Which of the following gives the solutions of the quadratic equation $2x^2 - 7x = 11$?

A. $x = \frac{7 \pm \sqrt{49 - 44}}{2}$ **C.** $x = \frac{-7 \pm \sqrt{49 + 88}}{4}$

B. $x = \frac{-7 \pm \sqrt{49 + 44}}{2}$ **D.** $x = \frac{7 \pm \sqrt{49 + 88}}{4}$

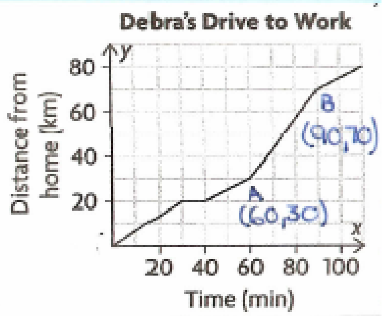
$2x^2 - 7x = 11$

$\hookrightarrow 2x^2 - 7x - 11 = 0$ {Standard form}

$a = 2,\ b = -7,\ c = -11$

10. The graph represents Debra's drive to work on a typical morning. What was Debra's speed on the fastest part of her journey?

A. 43 km/h **B.** 120 km/h **C.** 80 km/h **D.** 60 km/h



Debra was travelling the fastest from A to B.

Since speed $\Rightarrow$ slope: Speed $= \frac{70-30}{90-60}$

$= \frac{40}{30}$

$= 1.\overline{3}$ Km/min.

Convension: $\frac{1.\overline{3}\text{ Km}}{1\,\cancel{\text{min}}} \times \frac{60\,\cancel{\text{min}}}{1\text{h}} = 80$ Km/h

11. Two similar objects have a scale factor of $1:2.5$ associated with them. The volume of the smaller object is $400\ \text{cm}^3$. What is the larger object's volume?

A. $1000\ \text{cm}^3$ **B.** $2500\ \text{cm}^3$ **C.** $6250\ \text{cm}^3$ **D.** none of these

Volume of similar object $= K^3$(volume of original object)

$$= (2.5)^3(400\ \text{cm}^3)$$

$$= (15.625)(400)$$

$$= 6250\ \text{cm}^3$$

NUMERICAL RESPONSE

12. Snappy Cards makes two types of greeting cards: scratch'n'sniff, or with a recorded tune that plays when the card is open. The company can make at most 250 scratch'n'sniff cards and at most 175 tune cards per day. However, they can make 300 or more cards in total per day. Scratch'n'sniff cards cost \$1.25 to make, and tune cards cost \$2.00 to make. Let s represent the number of scratch'n'sniff cards and t represent the number of tune cards.

a) State the restrictions: $s \in$ W, $t \in$ W

b) State the constraints: $s \geq 0$, $s \leq 250$, $t \geq 0$, $t \leq 175$, $s + t \geq 300$

c) Define the objective function: cost $C = 1.25s + 2.00t$

13. Verify that the point $(-3, 3)$ is a solution to this system:

$\{(x, y) \mid 5y - x \geq 7, x \in R, y \in R\}$

$\{(x, y) \mid x + 2y < 5, x \in R, y \in R\}$

LS	RS
$5(3) - (-3)$ 18	7

LS	RS
$-3 + 2(3)$ 3	5

LS ≥ RS? ✓ LS < RS ✓

14. a) Complete the partial factoring of the function $f(x) = -3x^2 + 24x - 18$:

$$f(x) = \underline{-3}\,x(x \underline{-\ 8}) - \underline{18}$$

b) Determine the y-intercept: $(0, \underline{-18})$.

c) Determine another point with the same y-coordinate as the y-intercept:
$(\underline{8}, \underline{-18})$

$x = \dfrac{0+8}{2} = \dfrac{8}{2}$ or 4

d) Determine the equation of the axis of symmetry: $x = \underline{4}$.
Determine the vertex: $(\underline{4}, \underline{30})$.

↳ $f(x) = -3x^2 + 24x - 18$
$f(4) = -3(4)^2 + 24(4) - 18$
$f(4) = -3(16) + 96 - 18$
$f(4) = -48 + 96 - 18$
$f(4) = 30$

15. A quadratic function is given by $y = 4(x + 5)^2 - 35$. Determine

a) the coordinates of its vertex: $(\underline{-5}, \underline{-35})$ *Opens Upward

b) its y-intercept: $\underline{65}$ $y = 4(0+5)^2 - 35$

c) its domain and range: $\{(x, y) \mid \underline{x \in R, y \geq -35}, y \in R\}$

16. Solve the equation $2x^2 - 5x - 12 = 0$.

$x = \underline{4}$ or $x = \underline{-1.5}$

$a = 2, b = -5, c = -12$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(2)(-12)}}{2(2)}$$

$$x = \frac{5 \pm \sqrt{25 + 96}}{4}$$

$$x = \frac{5 \pm \sqrt{121}}{4}$$

$$x = \frac{5 \pm 11}{4}$$

$$x = \frac{5 + 11}{4} \quad \text{or} \quad x = \frac{5 - 11}{4}$$

$$x = \frac{16}{4} \qquad x = \frac{-6}{4}$$

$$x = 4 \qquad x = -1.5$$

17. Solve the equation $7x^2 - 2x + 1 = 3x + 2$. Give exact solutions.

$x = \dfrac{5+\sqrt{53}}{14}$ or $x = \dfrac{5-\sqrt{53}}{14}$

$7x^2 - 2x + 1 = 3x + 2$

$7x^2 - 2x - 3x + 1 - 2 = 0$

$7x^2 - 5x - 1 = 0$

$a = 7,\ b = -5,\ c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(7)(-1)}}{2(7)}$$

$$x = \frac{5 \pm \sqrt{25 + 28}}{14}$$

$$x = \frac{5 \pm \sqrt{53}}{14}$$

18. Nina and Franco are comparing their SUV and sedan fuel costs. On a 250 km trip, the SUV uses 45.3 L of gas. On a similar trip, of 180 km, the sedan uses 28.7 L of gas. Rounding to the nearest tenth, what is the fuel economy

a) of the SUV? 18.1 L/100 km

b) of the sedan? 15.9 L/100 km

c) By how much, to the nearest percent, is the sedan more fuel efficient? 12 %

a) SUV

$$\frac{45.3\,L}{250\,Km} = \frac{x}{100\,Km}$$

$$\frac{4530}{250} = \frac{\cancel{250}x}{\cancel{250}}$$

$$18.1\,L = x$$

b) SEDAN

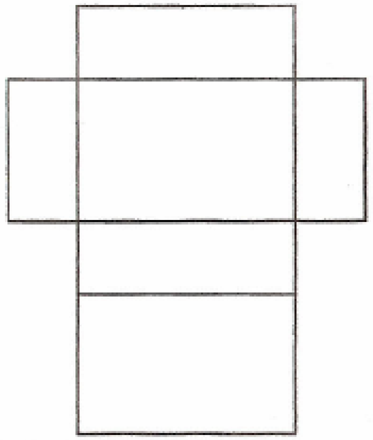
$$\frac{28.7\,L}{180\,Km} = \frac{x}{100\,Km}$$

$$\frac{2870}{180} = \frac{\cancel{180}x}{\cancel{180}}$$

$$15.9\,L = x$$

c) $\frac{15.9}{18.1} \times 100 = 88\%$ $100\% - 88\% = 12\%$

19. The scale diagram represents the net for a gift box. The base of the box is 15 mm wide on the diagram. The actual box will be 22.5 cm wide.



a) What is the scale ratio of the diagram? 1: 15 → (22.5 ÷ 1.5) 15 mm = 1.5 cm

b) How many times larger is the surface area of the actual box than the area of the diagram? 225 times larger $K^2 = (15)^2$ or 225.

21. Consider the system of linear inequalities:

$\{(x, y) \mid 2y - 5x < 7, x \in W, y \in W\}$

$\{(x, y) \mid x \geq 4, x \in W, y \in W\}$

a) Determine the boundary of each inequality, how it should be graphed, and which half plane (left or right) should be shaded.

Equations of boundaries:

→ $2y-5x=7$ (dashed)

→ $x=4$ (solid + stippled)

* Solution region stippled.

Two points on each boundary:

→ $2y-5x=7$

x-int:

$2(0)-5x=7$

$\frac{\cancel{-5}x}{\cancel{-5}}=\frac{7}{-5}$

$x=-1.4$

y-int:

$2y-5(0)=7$

$\frac{\cancel{2}y}{\cancel{2}}=\frac{7}{2}$

$y=3.5$

→ $x=4$

Vertical Line

Test Points:

→ $2y-5x<7$; Test Point $(0,0)$

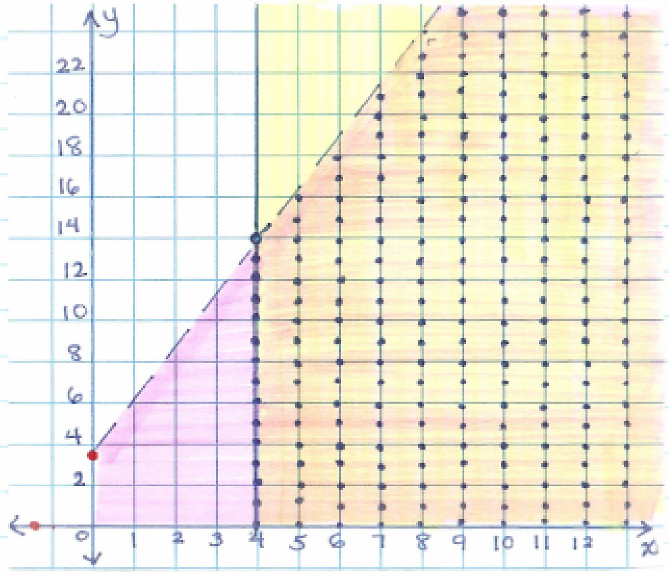
L.S	R.S
$2y-5x$	7
$2(0)-5(0)$	
0	

Since $0<7$, $(0,0)$ is located in the solution region.

→ $x \geq 4$

Shaded to the right of the line.

GRAPH:



b) Identify two points in the solution set, and validate them as solutions.

2 points in the solution set:

(6, 4)

→ $2y-5x<7$

L.S	R.S
$2(4)-5(6)$	7
$8-30$	
-22	

$-22<7$ ✓

→ $x \geq 4$

L.S	R.S
6	4

$6 \geq 4$ ✓

(12, 10)

→ $2y-5x<7$

L.S	R.S
$2(10)-5(12)$	7
$20-60$	
-40	

$-40<7$ ✓

→ $x \geq 4$

L.S	R.S
12	4

$12 \geq 4$ ✓

22. Lorel and Shaida are making shakes and smoothies for the drinks stand at their school fair. They can make a maximum of 280 drinks altogether, of which at most 120 can be shakes. They plan to sell shakes for \$3.50 each and smoothies for \$2.75 each.

a) Create an algebraic model of the situation.

Let x represent the number of shakes.
Let y represent the number of smoothies.
Let R represent Revenue.

Restrictions:
$x \in W$
$y \in W$

Constraints: $x \geq 0$, $y \geq 0$
$x + y \leq 280$
$x \leq 120$

Objective Function: $R = 3.50x + 2.75y$

b) Graph the system of inequalities in your model on the grid provided. State the vertices of the feasible region.

Equations of boundaries:

→ $x+y=280$ (Solid + stippled)

→ $x=120$ (solid + stippled)

Two points on each boundary:

→ $x+y=280$

x-int:

$x+0=280$

$x=280$

y-int:

$0+y=280$

$y=280$

→ $x=120$

Vertical Line

Test Points:

→ $x+y \leq 280$; Test Point $(0,0)$

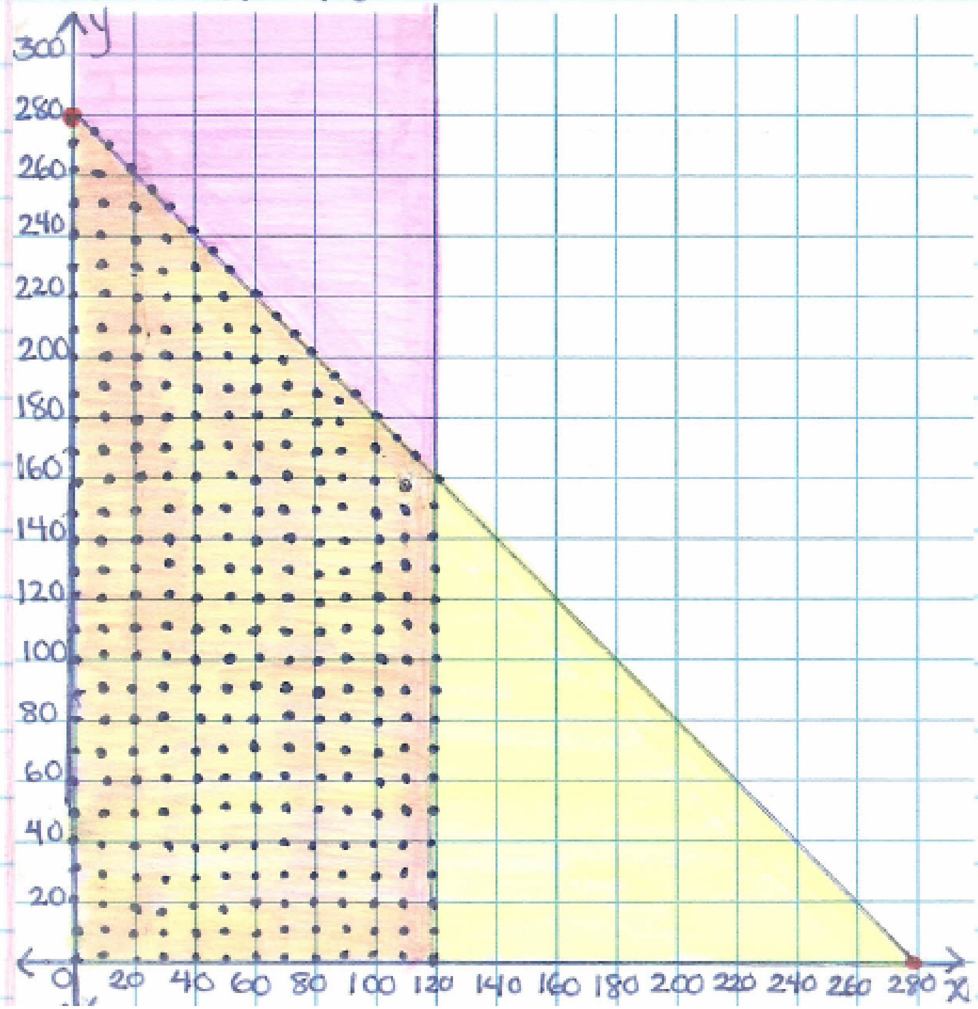
L.S | R.S.
$0+0$ | 280
0 |

Since $0 \leq 280$, $(0,0)$ is located in the solution region.

→ $x \leq 120$

Shaded to the left of the line.

GRAPH:



Vertices of Feasible Region:

(0,0), (0,280), (120,160), (120,0)

c) Predict which point in the feasible region maximizes the objective function. Explain.

For (0,0): $R = 3.50x + 2.75y$
$R = 3.50(0) + 2.75(0)$
$R = \$0$

For (0,280): $R = 3.50x + 2.75y$
$R = 3.50(0) + 2.75(280)$
$R = \$770$

For (120,160): $R = 3.50x + 2.75y$
$R = 3.50(120) + 2.75(160)$
$R = \$860$ *(Maximum Revenue)

For (120,0): $R = 3.50x + 2.75y$
$R = 3.50(120) + 2.75(0)$
$R = \$420$.

d) Determine the maximum revenue that Lorel and Shaida can make. Verify that your solution satisfies the constraints of the original problem.

Maximum Revenue => $860.

Verifying Solution:

(x,y) is $(120,160)$

$x + y \le 280$

L.S	R.S
$x+y$	280
$120+160$	
280	

$280 \le 280$

Valid

(x,y) is $(120,160)$

$x \le 120$

L.S	R.S.
x	120
120	

$120 \le 120$.

Valid

24. A parabola has vertex (3, −8.5) and y-intercept −4.

h k

a) Write the equation of the parabola in vertex form. Show your work.

$y = a(x-h)^2 + k$

$y = a(x-3)^2 - 8.5$

$\Rightarrow y = 0.5(x-3)^2 - 8.5$

To determine "a":

$y = a(x-3)^2 - 8.5$

$-4 = a(0-3)^2 - 8.5$

$-4 = a(-3)^2 - 8.5$

$-4 = a(9) - 8.5$

$-4 = 9a - 8.5$

$-4 + 8.5 = 9a$

$\frac{4.5}{9} = \frac{\cancel{9}a}{\cancel{9}}$ $\quad 0.5 = a$

b) Write the equation of the parabola in standard form.

$y = 0.5(x-3)^2 - 8.5$

$y = 0.5(x-3)(x-3) - 8.5$

$y = 0.5(x^2 - 3x - 3x + 9) - 8.5$

$y = 0.5(x^2 - 6x + 9) - 8.5$

$y = 0.5x^2 - 3x + 4.5 - 8.5$

$y = 0.5x^2 - 3x - 4$

c) Determine the exact values of the x-intercepts of the parabola.

$y = 0.5x^2 - 3x - 4$

$a = 0.5,\ b = -3,\ c = -4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(0.5)(-4)}}{2(0.5)}$$

$$x = \frac{3 \pm \sqrt{9 + 8}}{1}$$

$$x = 3 \pm \sqrt{17}$$

26. Determine a quadratic equation in the form $ax^2 + bx + 30 = 0$ that has roots $x = 5$ and $x = -3$.

For Example: $(x-5)(x+3) = 0$

$x^2 + 3x - 5x - 15 = 0$

$x^2 - 2x - 15 = 0$

28. Murray is wrapping a number of identical boxes for gifts for co-workers. Each box needs a length of 57 cm from a roll of gift wrap. There are two gift-wrap options, at different stores: 3 m rolls at \$16.49 or 4 m rolls at \$19.99. To save on shopping time, Murray wants to get only one kind of wrap.

a) If Murray has 15 gifts to wrap, should he buy 3 m or 4 m rolls?

$15 \times 57\text{cm} = 855\text{cm}$

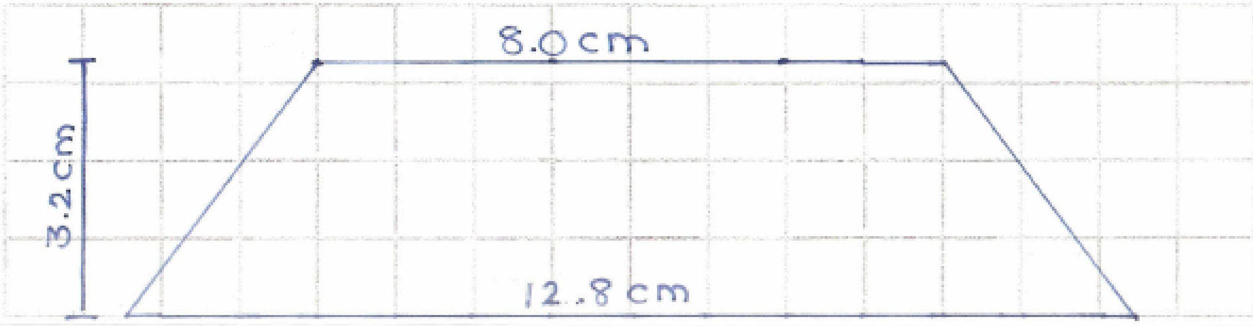
$855\cancel{\text{cm}} \times \frac{1\text{m}}{100\cancel{\text{cm}}} = 8.55\text{m}$

Since he requires 8.55m in total, he should buy 3 of the 3m rolls. This would cost $3 \times \$16.49 = \$49.47$

b) If Murray has many extra gifts to wrap, which rolls should he buy?

If Murray purchased 3 of the 3m rolls, he would not be able to wrap any extra gifts (only 45cm left) He should therefore purchase 3 of the 4m rolls.

29. An isosceles trapezoid has parallel sides measuring 6.4 cm and 4.0 cm and a height of 1.6 cm. Draw a 2:1 scale diagram of the trapezoid.



30. Marie is making table runners from a 1 : 12 scale plan. Her plan measures 2.4 cm by 28.2 cm. How much material, to the nearest tenth of a square metre, does Marie need for 8 runners?

Area $= 2.4\text{cm} \times 28.2\text{cm}$
$= 67.68\text{cm}^2$

Area of similar 2-D shape $= K^2$(Area of Original Shape).
$= (12)^2(67.68\text{cm}^2)$
$= (144)(67.68\text{cm}^2)$
$= 9745.92\text{ cm}^2$
or 0.9746 m^2

$0.9746\text{m}^2 \times 8 \doteq 7.8\text{ m}^2$
Marie requires approximately 7.8m^2 of material.