

SOLUTIONS \Rightarrow EXPONENTIAL GROWTH REVIEW

1. $\{(1, 2), (2, 4), (3, 6), (4, 8), \dots\}$

\hookrightarrow

| x | y |
|-----|-----|
| 1 | 2 |
| 2 | 4 |
| 3 | 6 |
| 4 | 8 |

- x -values increase by 1.
- y -values increasing by $+2$, therefore these coordinates are linear.

2. $\{(0, 1), (1, 5), (2, 25), (3, 125), \dots\}$

\hookrightarrow

| x | y |
|-----|-----|
| 0 | 1 |
| 1 | 5 |
| 2 | 25 |
| 3 | 125 |

- x -values increase by 1.
- y -values are increasing by a factor of 5 ($\times 5$); therefore these coordinates are exponential.

3. $\{(-1, 3), (0, 6), (1, 9), (2, 12), \dots\}$

↳

| x | y |
|----|----|
| -1 | 3 |
| 0 | 6 |
| 1 | 9 |
| 2 | 12 |

- x-values increase by 1.
- y-values are increasing by +3, therefore these coordinates are linear.

2a) $(3, 12, 48, 192, 768, 3072)$

Common ratio is 4.

b) $(6, 3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16})$

Common ratio is $\frac{1}{2}$

c) $(7, -21, 63, -189, 567, -1701)$

Common ratio is -3.

- 3 a) $y = 2^x$ Common ratio is 2.
b) $y = 5^x$ Common ratio is 5.
c) $y = (0.6)^x$ Common ratio is 0.6.
d) $y = 100(0.6)^x$ Common ratio is 0.6.
e) $y = 200(1.02)^x$ Common ratio is 1.02.

4. Investment: \$1500 Interest: 5% per year.

a) $y = 1500(1.05)^x$

b) After 7 years $\Rightarrow x = 7$

$$y = 1500(1.05)^7$$

$$\hat{y} = \$2110.65$$

5.

$$y = 1000(2)^{\frac{x}{1}} \rightarrow \text{Time it takes to double.}$$

\downarrow \downarrow
Initial Value Doubles.

a). After 4 min $\Rightarrow x = 4$.

$$\begin{aligned} y &= 1000(2)^4 \\ &= 1000(16) \\ &= 16000 \end{aligned}$$

16 000 bacteria after 4 min.

b) After 6 min $\Rightarrow x = 6$.

$$\begin{aligned}y &= 1000 (2)^6 \\ &= 1000 (64) \\ &= 64\,000.\end{aligned}$$

64 000 bacteria after 6 min.

c) After 10 min $\Rightarrow x = 10$.

$$\begin{aligned}y &= 1000 (2)^{10} \\ &= 1000 (1024) \\ &= 1\,024\,000\end{aligned}$$

1 024 000 bacteria after 10 min.

$$6. A = P \left(\frac{1}{2} \right)^{\frac{t}{92}}$$

a) $\overset{\sim}{\text{Initial Amount}} \Rightarrow P = 7\text{g}$
 $\text{Time} \Rightarrow t = 40\text{h}$

$$A = 7 \left(\frac{1}{2} \right)^{\frac{40}{92}}$$

$$A = 7 (0.5)^{\frac{40}{92}}$$

$$A \approx 5.2\text{g}$$

b) According to the equation given, the half-life of radon-222 is 92 hours.

7. EQUATION: $y = 225\,000(1.10)^{\frac{x}{5}}$

WHEN $x=12$ $y = 225\,000(1.10)^{\frac{12}{5}}$
 $y = \$282\,829.67$

8. a) 5^{-2} b) 7^0 c) 4^{-1} d) 2^{-3} e) $\left(\frac{1}{2}\right)^{-4}$
 $= \frac{1}{5^2}$ $= 1$ $= \frac{1}{4^1}$ $= \frac{1}{2^3}$ $= 2^4$
 $= \frac{1}{25}$ $= 1$ $= \frac{1}{4}$ $= \frac{1}{8}$ $= 16$

f) $(-12)^0$ g) $\left(-\frac{2}{5}\right)^{-4}$ h) -8^0 i) $\left(-\frac{4}{3}\right)^{-3}$
 $= 1$ $= \left(-\frac{5}{2}\right)^4$ $= -1$ $= \left(-\frac{3}{4}\right)^3$
 $= \frac{625}{16}$ $= \frac{-27}{64}$

$$j) 2^{-2} + 3^{-2} \quad k) \left(\frac{3m}{n^2} \right)^{-3}$$

$$= \frac{1}{2^2} + \frac{1}{3^2}$$

$$= \frac{1}{4} + \frac{1}{9}$$

$$= \frac{9}{36} + \frac{4}{36}$$

$$= \frac{13}{36}$$

$$= \left(\frac{n^2}{3m} \right)^3$$

$$= \frac{n^6}{27m^3}$$

$$9a) 3^{x+2} = 3^{2x+6}$$

$$x+2 = 2x+6$$

$$2-6 = 2x-x$$

$$-4 = x$$

$$b) 2^{2x+2} = 16^{x+6}$$

$$2^{2x+2} = (2^4)^{x+6}$$

$$2^{2x+2} = 2^{4x+24}$$

$$2x+2 = 4x+24$$

$$2-24 = 4x-2x$$

$$\frac{-22}{2} = \frac{2x}{2}$$

$$-11 = x$$

$$c) 2^{2x+2} = 16(2^x)$$

$$2^{2x+2} = (2^4)(2^x)$$

$$2^{2x+2} = 2^{4+x}$$

$$2x+2 = 4+x$$

$$2x-x = 4-2$$

$$x = 2$$

$$d) 8^x + 24 = 88$$

$$8^x = 88 - 24$$

$$8^x = 64$$

$$8^x = 8^2$$

$$x = 2$$

$$e) 3^{x-2} = \frac{27^{2x}}{9^{x-1}}$$

$$3^{x-2} = \frac{(3^3)^{2x}}{(3^2)^{x-1}}$$

$$3^{x-2} = \frac{3^{6x}}{3^{2x-2}}$$

$$3^{x-2} = 3^{6x-(2x-2)}$$

$$3^{x-2} = 3^{6x-2x+2}$$

$$3^{x-2} = 3^{4x+2}$$

$$x-2 = 4x+2$$

$$-2-2 = 4x-x$$

$$\frac{-4}{3} = \frac{\cancel{3}x}{\cancel{3}}$$

$$\frac{-4}{3} = x$$

$$f) (4^x)(2^{x+3}) = 16^{2x-5}$$

$$(2^2)^x (2^{x+3}) = (2^4)^{2x-5}$$

$$(2^{2x})(2^{x+3}) = 2^{8x-20}$$

$$2^{2x+x+3} = 2^{8x-20}$$

$$2^{3x+3} = 2^{8x-20}$$

$$3x+3 = 8x-20$$

$$3+20 = 8x-3x$$

$$\frac{23}{5} = \frac{5x}{5}$$

$$\frac{23}{5} = x$$

$$10. a) 25^{1/2} \quad b) 125^{2/3} \quad c) (-64)^{3/2}$$

$$\begin{aligned} &= (\sqrt{25})^1 \\ &= (5)^1 \\ &= 5 \end{aligned}$$

$$\begin{aligned} &= (\sqrt[3]{125})^2 \\ &= (5)^2 \\ &= 25 \end{aligned}$$

$$= (\sqrt{-64})^3$$

NO REAL SOLUTION

$$\begin{aligned} d) -81^{3/4} \\ &= -(\sqrt[4]{81})^3 \\ &= -(3)^3 \\ &= -27 \end{aligned}$$

$$\begin{aligned} e) \left(\frac{8}{27}\right)^{-3/2} \\ &= \left(\frac{27}{8}\right)^{3/2} \\ &= \left(\sqrt[3]{\frac{27}{8}}\right)^2 \\ &= \left(\frac{3}{2}\right)^2 \\ &= \frac{9}{4} \end{aligned}$$

$$11. \text{ a) } \sqrt{17} \quad \text{b) } (\sqrt[6]{8})^2 \quad \text{c) } \frac{1}{2\sqrt{x}}$$

$$= 17^{\frac{1}{2}}$$

$$= 8^{\frac{2}{6}}$$

$$= \frac{1}{2x^{\frac{1}{2}}}$$

$$\text{d) } (\sqrt{x})^4$$

$$\text{e) } (\sqrt[4]{25})^3$$

$$= (x^{\frac{1}{2}})^4$$

$$= 25^{\frac{3}{4}}$$

$$= x^{\frac{4}{2}}$$

$$= x^2$$

12. The graph of $y = b^x$ represents exponential growth when $b > 1$.

13. The graph of $y = b^x$ represents exponential decay when $0 < b < 1$.

14. $y = b^x$

Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \mid y > 0, y \in \mathbb{R}\}$

y-int: $y = 1$ OR $(0, 1)$

Location of Horizontal Asymptote: $y = 0$
OR
x-axis.

$$15. a) 3^4 = 81$$

$$b) 3^{-3} = \frac{1}{27}$$

$$\hookrightarrow \log_3 81 = 4$$

$$\hookrightarrow \log_3 \left(\frac{1}{27} \right) = -3$$

$$16. \text{ a) } \log_3 27 = 3$$

$$\hookrightarrow 3^3 = 27$$

$$\text{ b) } \log_8 2 = \frac{1}{3}$$

$$\hookrightarrow 8^{\frac{1}{3}} = 2$$

$$17. \text{ a) } \log_3 x = 5$$

$$3^5 = x$$

$$243 = x$$

$$\text{ b) } \log_2 (x-3) = 2$$

$$2^2 = x-3$$

$$4 = x-3$$

$$4+3 = x$$

$$7 = x$$

$$\text{ c) } \log_3 (-x+1) = 6$$

$$3^6 = -x+1$$

$$729 = -x+1$$

$$729 - 1 = -x$$

$$728 = -x$$

$$-728 = x$$

18. a) $\log_2 8$

$$x = \log_2 8$$

$$2^x = 8$$

$$2^x = 2^3$$

$$x = 3$$

b) $\log_5 \frac{1}{25}$

$$x = \log_5 \frac{1}{25}$$

$$5^x = \frac{1}{25}$$

$$5^x = \frac{1}{5^2}$$

$$5^x = 5^{-2}$$

$$x = -2$$

c) $\log_{\frac{1}{3}} \frac{1}{243}$

$$x = \log_{\frac{1}{3}} \frac{1}{243}$$

$$\left(\frac{1}{3}\right)^x = \frac{1}{243}$$

$$\frac{1}{3^x} = \frac{1}{3^5}$$

$$(3^{-1})^x = 3^{-5}$$

$$3^{-1x} = 3^{-5}$$

$$\frac{-1x}{-1} = \frac{-5}{-1}$$

$$x = 5$$

$$19. \quad 2 \log_5 3 + \log_5 6 - \log_5 27$$

$$= \log_5 3^2 + \log_5 6 - \log_5 27$$

$$= \log_5 9 + \log_5 6 - \log_5 27$$

$$= \log_5 (9 \cdot 6) - \log_5 27$$

$$= \log_5 54 - \log_5 27$$

$$= \log_5 \left(\frac{54}{27} \right)$$

$$= \log_5 2$$

$$20. \frac{1}{2} \log_2 16 + \log_2 8 - \log_2 4$$

Evaluate each term separately:

$$\frac{1}{2} \log_2 16 = x$$

$$\log_2 16^{\frac{1}{2}} = x$$

$$\log_2 4 = x$$

$$2^x = 4$$

$$2^x = 2^2$$

$$x = 2$$

$$\log_2 8 = x$$

$$2^x = 8$$

$$2^x = 2^3$$

$$x = 3$$

$$\log_2 4 = x$$

$$2^x = 4$$

$$2^x = 2^2$$

$$x = 2$$

Therefore we have:

$$\begin{aligned} &= 2 + 3 - 2 \\ &= 5 - 2 \\ &= 3 \end{aligned}$$

