Solutions => Quadratics Exam Review

1.

a) 4, 26, 84, 196, 380

D_1 22 58 112 184

D_2 36 54 72

D_3 18 18 Cubic

b) (1,5), (2,15), (3,45), (4,135), (5,405)

y-values: 5, 15, 45, 135, 405

×3 ×3 ×3 ×3 Exponential

c) -10, -2, 13, 35, 64, 100

D_1 8 15 22 29 36

D_2 7 7 7 7 QUADRATIC

d) (0,1000) (1, 995.1) (2, 980.4) (3, 955.9) (4, 921.6) (5, 877.5)

y-values: 1000, 995.1, 980.4, 955.9, 921.6, 877.5

D_1 -4.9 -14.7 -24.5 -34.3 -44.1

D_2 -9.8 -9.8 -9.8 -9.8

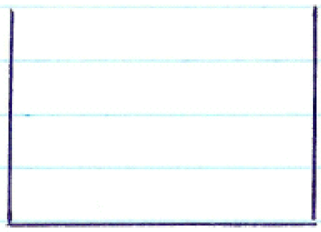
QUADRATIC

e) $-11, 21, 53, 85$

$\quad\ \vee \quad \vee \quad \vee$

D, $\quad 32 \quad 32 \quad 32$ $\qquad$ LINEAR

2. Perimeter $= 300$m



Let $x =$ width

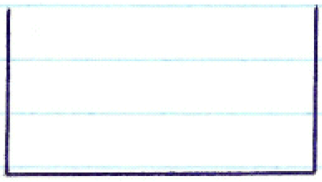
Then $300 - 2x =$ length

$A =$ length $\times$ width

$A = (300 - 2x)(x)$

<u>OR</u>

$A = (300 - 2w)(w)$

3. 

$P = 688\,m$

Let x = width

Then $\frac{688-2x}{2}$ = length

$344 - x$ = length

A = length $\times$ width

$A = (344-x)(x)$

4.a) $y = (x+3)^2 - 7$ (SF)

$y = (x+3)(x+3) - 7$

$y = x^2 + 3x + 3x + 9 - 7$

$y = x^2 + 6x + 2$ (GF)

b) $5(y - 1/3) = (x-8)^2$ (TF)

$5(y - 1/3) = (x-8)(x-8)$

$5(y - 1/3) = x^2 - 8x - 8x + 64$

$5(y - 1/3) = x^2 - 16x + 64$

$y - 1/3 = 1/5(x^2 - 16x + 64)$

$y - 1/3 = 1/5x^2 - 16/5x + 64/5$

$y = 1/5x^2 - 16/5x + 64/5 + 1/3$

$y = 1/5x^2 - 16/5x + 192/15 + 5/15$

$y = 1/5x^2 - 16/5x + 197/15$ (GF)

c) $2(y-5)=(x+7)^2$ (TF)

$2(y-5)=(x+7)(x+7)$
$2(y-5)=x^2+7x+7x+49$
$2(y-5)=x^2+14x+49$
$y-5=\frac{1}{2}(x^2+14x+49)$
$y-5=\frac{1}{2}x^2+\frac{14}{2}x+\frac{49}{2}$
$y=\frac{1}{2}x^2+7x+\frac{49}{2}+\frac{5}{1}$
$y=\frac{1}{2}x^2+7x+\frac{49}{2}+\frac{10}{2}$
$y=\frac{1}{2}x^2+7x+\frac{59}{2}$ (GF)

d) $y=2(x+3)^2+7$ (SF)
$y=2(x+3)(x+3)+7$
$y=(2x+6)(x+3)+7$
$y=2x^2+6x+6x+18+7$
$y=2x^2+12x+25$ (GF)

5\. a) $\frac{1}{2}y = (x+7)^2$ (TF) b) $3(y-7) = (x-9)^2$ (TF)

$y = 2(x+7)^2$ (SF)

$y - 7 = 1/3(x-9)^2$

$y = 1/3(x-9)^2 + 7$ (SF)

c) $7(y+8) = (x+2)^2$ (TF)

$y + 8 = 1/7(x+2)^2$

$y = 1/7(x+2)^2 - 8$ (SF)

6\. $h = -5t^2 + 20t$

a) $t = 3$

$h = -5(3)^2 + 20(3)$

$h = -5(9) + 60$

$h = -45 + 60$

$h = 15$ m

b) $h = -5t^2 + 20t$

(2) $h = -5(t^2 - 4t)$

(3) $h - 20 = -5(t^2 - 4t + 4)$

(4) $h - 20 = -5(t-2)^2$

(5) $h = -5(t-2)^2 + 20$

Vertex $(2, 20)$

Time Height

The maximum height of the rocket is 20m

It takes 2 seconds to reach the maximum height.

c) When the rocket hits the ground → $h=0$.

$0=-5t^2+20t$

3 Methods to solve this problem.

#1 Factoring

$0=-5t^2+20t$

$0=-5t(t-4)$

$\frac{\cancel{-5}t}{\cancel{-5}}=\frac{0}{-5}$ or $t-4=0$

$t=0$ $\qquad$ $t=4$

It took to rocket 4 seconds to hit the ground.

#2 Quadratic Formula

$0 = -5t^2 + 20t$

$a = -5,\ b = 20,\ c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-20 \pm \sqrt{(20)^2 - 4(-5)(0)}}{2(-5)}$

$x = \frac{-20 \pm \sqrt{400 - 0}}{-10}$

$x = \frac{-20 \pm \sqrt{400}}{-10}$

$x = \frac{-20 \pm 20}{-10}$

$x = \frac{-20 + 20}{-10}$ $\qquad$ $x = \frac{-20 - 20}{-10}$

$x = \frac{0}{-10}$ $\qquad$ $x = \frac{-40}{-10}$

$x = 0$ $\qquad$ $x = 4$

#3 Completing the Square

$0 = -5t^2 + 20t$

(2) $0 = t^2 - 4t$

(3) $4 = t^2 - 4t + 4$

(4) $4 = (t-2)^2$

(5) $\pm\sqrt{4} = t - 2$

(6) $\pm 2 = t - 2$

$2 \pm 2 = t$

$2 + 2 = t$ <u>OR</u> $2 - 2 = t$

$4 = t$ $\qquad$ $0 = t$

7. $h = -5t^2 + 5t + 3$

a) The diving board is 3 m high.
* The constant term in the equation represents initial height.

b) $h = -5t^2 + 5t + 3$

① $h - 3 = -5t^2 + 5t$
② $h - 3 = -5(t^2 - 1t)$
③ $h - 3 - \frac{5}{4} = -5(t^2 - 1t + 1/4)$

④ $h - \frac{12}{4} - \frac{5}{4} = -5(t - 1/2)^2$
$h - 17/4 = -5(t - 1/2)^2$

⑤ $h = -5(t - 1/2)^2 + 17/4$
Vertex $(1/2, 17/4)$

She reaches a maximum height of $17/4$ m (4.25 m) after $1/2$ (0.5) seconds.

d) $0 = -5t^2 + 5t + 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(-5)(3)}}{2(-5)}$$

$$x = \frac{-5 \pm \sqrt{25 + 60}}{-10}$$

$$x = \frac{-5 \pm \sqrt{85}}{-10}$$

$$x = \frac{-5 \pm 9.22}{-10}$$

$$x = \frac{-5 + 9.22}{-10}$$

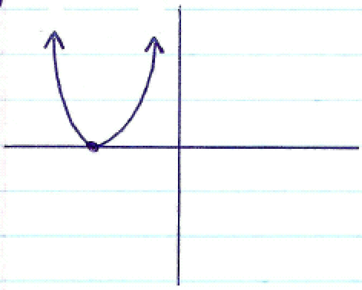
$x = -\cancel{0.422}$ seconds

$$x = \frac{-5 - 9.22}{-10}$$

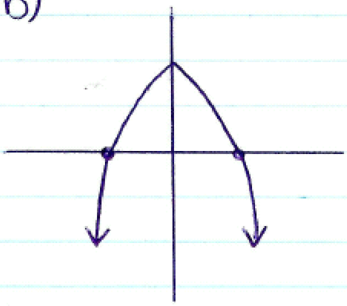
$x = 1.422$ seconds

8.

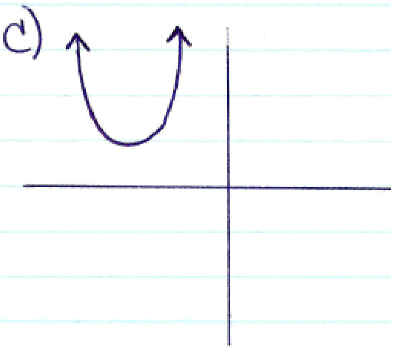
a)



b)



c)



9. One real root $\Rightarrow D=0$

$y = x^2 - 3x + K$

$a=1,\ b=-3,\ c=K$

$D = b^2 - 4ac$

$0 = (-3)^2 - 4(1)(K)$

$0 = 9 - 4K$

$\frac{\cancel{4}K}{\cancel{4}} = \frac{9}{4}$

$K = \frac{9}{4}$

10. 2 real roots $\Rightarrow D > 0$

$y = x^2 - 8x + K$

$a = 1,\ b = -8,\ c = K$

$D = b^2 - 4ac$

$b^2 - 4ac > 0$

$(-8)^2 - 4(1)(K) > 0$

$64 - 4K > 0$

$\frac{-\cancel{4}K}{-\cancel{4}} > \frac{-64}{-4}$

$K < 16$

11. a) $t_n = 3n^3 + 2n^2 - 4$

$t_1 = 3(1)^3 + 2(1)^2 - 4$
$t_1 = 3(1) + 2(1) - 4$
$t_1 = 3 + 2 - 4$
$t_1 = 1$

$t_2 = 3(2)^3 + 2(2)^2 - 4$
$t_2 = 3(8) + 2(4) - 4$
$t_2 = 24 + 8 - 4$
$t_2 = 32 - 4$
$t_2 = 28$

$t_3 = 3(3)^3 + 2(3)^2 - 4$
$t_3 = 3(27) + 2(9) - 4$
$t_3 = 81 + 18 - 4$
$t_3 = 95$

$\{1, 28, 95\}$

b) $t_n = n^2 + 4n - 8$

$t_1 = (1)^2 + 4(1) - 8$

$t_1 = 1 + 4 - 8$

$t_1 = -3$

$t_2 = (2)^2 + 4(2) - 8$

$t_2 = 4 + 8 - 8$

$t_2 = 4$

$t_3 = (3)^2 + 4(3) - 8$

$t_3 = 9 + 12 - 8$

$t_3 = 13$

$\{-3, 4, 13\}$

12. $\frac{1}{a}(y-K) = (x-h)^2$

a) Up/Down => "K"

b) Left / Right => "h"

c) become wider or narrower => "a"