Geometric Sequences

Definition:

A geometric sequence is a pattern of numbers in which each term is found by "multiplying" the preceding term by the same amount.

3, 6, 12, 24, 48

Example 1:

$r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3}$

(5, 15, 45, 135, ...)

$= \frac{15}{5} = \frac{45}{15} = \frac{135}{45} = 3$

➡ **Each term is 3 times the preceding term.**

$r = 3$

Example 2:

(135, 45, 15, 5, ...)

$r = \frac{45}{135} = \frac{15}{45} = \frac{5}{15} =$

➡ **Each term is $\frac{1}{3}$ of the preceding term.**

$r = \frac{1}{3}$

Equation:

The following equation can be used to write the function of a geometric sequence...

$$t_n = t_1 r^{n-1}$$

t_1 = first term (a)

r = common multiplier / common ratio

n = the value of n / # of terms

Example 1:

Find t_{15} for the sequence {-2, 6, -18, 54,...}

Solution:

$r = \frac{6}{-2} = \frac{-18}{6} = \frac{54}{-18} = -3$

$t_1 = -2$

$r = -3$

$n = 15$

$t_n = t_1 r^{n-1}$

$t_{15} = (-2)(-3)^{15-1}$

$t_{15} = (-2)(-3)^{14}$

$t_{15} = (-2)(4\ 782\ 969)$

$t_{15} = -9\ 565\ 938$

Example 2:

Find the number of terms "n" for the following sequence:

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots, \frac{1}{256}$

Solution:

$t_1 = \frac{1}{2}$

$r = \frac{1}{2}$

$t_n = \frac{1}{256}$

$t_n = t_1 r^{n-1}$

$\frac{1}{256} = \frac{1}{2}\left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{256} \div \frac{1}{2} = \left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{256} \times \frac{2}{1} = \left(\frac{1}{2}\right)^{n-1}$

$\frac{2}{256} = \left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{128} = \left(\frac{1}{2}\right)^{n-1}$ We need to make the "bases" the same!

$\left(\frac{1}{2}\right)^7 = \left(\frac{1}{2}\right)^{n-1}$ Now that the "bases" are equal ...

$7 = n - 1$

$7 + 1 = n$

$8 = n$

* $\frac{\log(1/128)}{\log(1/2)}$

Don't forget that (any number)^0 = 1!!!

			t_1	d/r
①	12, 9, 6, 3, 0	arithmetic	12	$d=-3$

$d = 9-12$
$= -3$

②	12, 6, 3, 1.5, 0.75	geometric	12	$r=\frac{1}{2}$

$r=\frac{6}{12}=\frac{1}{2}$