



Geometric Sequences



Definition:

A geometric sequence is a pattern of numbers in which each term is found by "multiplying" the preceding term by the same amount.

3, 6, 12, 24, 48

Example 1:

$$(5, 15, 45, 135, \dots) \quad r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = \frac{15}{5} = \frac{45}{15} = \frac{135}{45} = 3$$

➡ Each term is **3** times the preceding term.
 $r = 3$

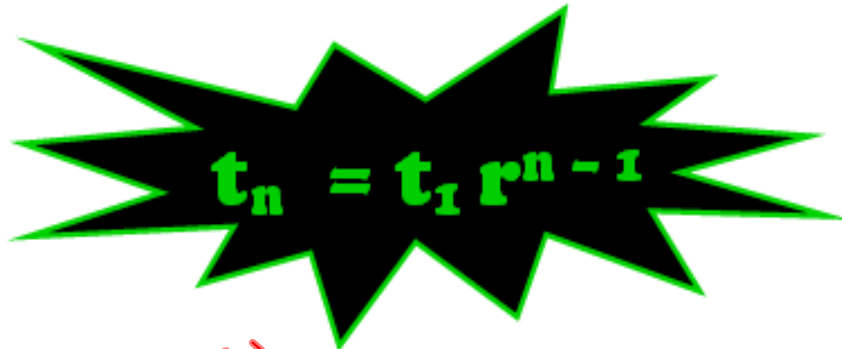
Example 2:

$$(135, 45, 15, 5, \dots) \quad r = \frac{45}{135} = \frac{15}{45} = \frac{5}{15} = \frac{1}{3}$$

➡ Each term is **$\frac{1}{3}$** of the preceding term.
 $r = \frac{1}{3}$

Equation:

The following equation can be used to write the function of a geometric sequence...


$$t_n = t_1 r^{n-1}$$

t_1 = first term (a)

r = common multiplier / common ratio

n = the value of n / \neq of terms

Example 1:

Find t_{15} for the sequence $\{-2, 6, -18, 54, \dots\}$

Solution:

$$r = \frac{6}{-2} = \frac{-18}{6} = \frac{54}{-18} = -3$$

$$t_1 = \underline{-2}$$

$$r = -3$$

$$n = 15$$

$$t_n = t_1 r^{n-1}$$

$$t_{15} = (-2)(-3)^{15-1}$$

$$t_{15} = (-2)(-3)^{14}$$

$$t_{15} = (-2)(4\,782\,969)$$

$$t_{15} = -9\,565\,938$$

Example 2:

Find the number of terms "n" for the following sequence:

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots, \frac{1}{256}$$

Solution:

$$\begin{aligned} t_1 &= \frac{1}{2} & t_n &= t_1 r^{n-1} \\ r &= \frac{1}{2} & \frac{1}{256} &= \frac{1}{2} \left(\frac{1}{2}\right)^{n-1} \\ t_n &= \frac{1}{256} & \frac{1}{256} \div \frac{1}{2} &= \frac{1}{2}^{n-1} \\ & & \frac{1}{256} \times 2 &= \frac{1}{2}^{n-1} \end{aligned}$$

$$* \frac{\log\left(\frac{1}{128}\right)}{\log\left(\frac{1}{2}\right)}$$

$$\begin{aligned} \frac{2}{256} &= \frac{1}{2}^{n-1} \\ \frac{1}{128} &= \frac{1}{2}^{n-1} && \text{We need to make the "bases" the same!} \\ \frac{1}{2} &= \frac{1}{2}^{n-1} \\ \left(\frac{1}{2}\right)^7 &= \frac{1}{2}^{n-1} && \text{Now that the "bases" are equal ...} \\ 7 &= n - 1 \\ 7 + 1 &= n \\ 8 &= n \end{aligned}$$

Don't forget that (any number)⁰ = 1!!!



① 12, 9, 6, 3, 0 arithmetic 12 $d = -3$

$d = 9 - 12$
 $= -3$

② 12, 6, 3, 1.5, 0.75 geometric 12 $r = \frac{1}{2}$

$r = \frac{6}{12} = \frac{1}{2}$