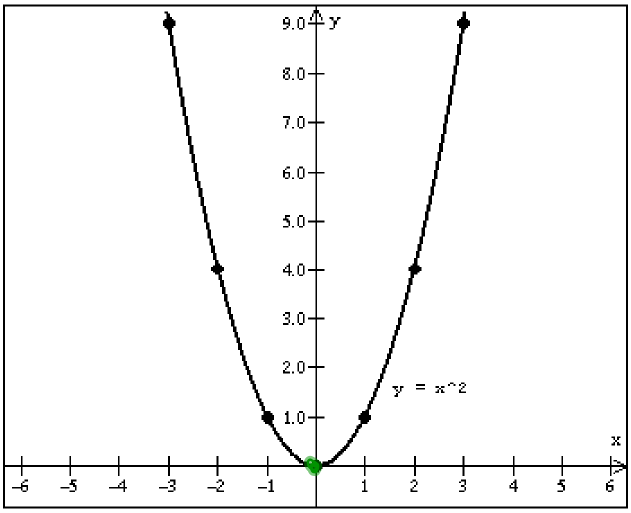
Graph of the Quadratic Function $f(x) = x^2$

$y = x^2$

Here are some points on the curve $y = x^2$:

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

Notice that the vertex is at (0, 0).
Notice that the graph "opens up", and therefore has a minimum value of (0, 0).
Notice that the y-axis (eqn: $x = 0$) is the axis of symmetry.



The graph of every quadratic function can be obtained by transforming the graph of $y = x^2$ **with**:

1. a vertical shift, (up / down)
2. a horizontal shift, (left / Right)
3. a reflection about the x-axis, (opens up / opens down)
4. and/or a vertical stretch or compression

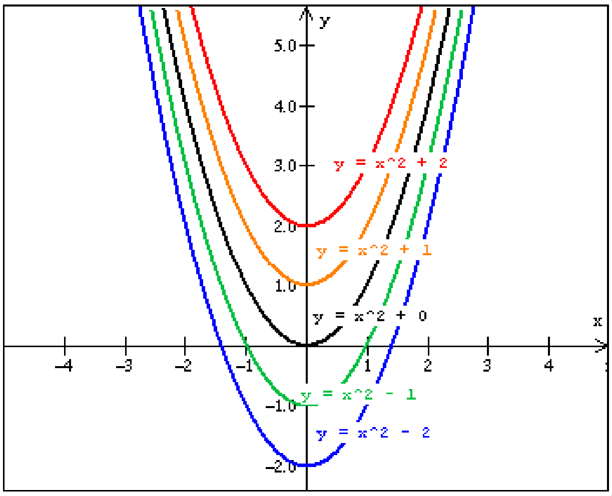
Graph of the Quadratic Function $f(x) = x^2 + k$ (a vertical shift):

Adding a constant to the graph of $y = x^2$ has the effect of shifting the graph vertically (up/down).

The graph of $y = x^2 + k$ is the graph of $y = x^2$ shifted ***up*** by k units when k is a positive number.

The graph of $y = x^2 + k$ is the graph of $y = x^2$ shifted ***down*** by k units when k is a negative number.

Notice that the vertex is at $(0, k)$.
Notice that the graph still "opens up", and has a minimum value.
Notice that the y-axis (eqn: $x = 0$) is still the axis of symmetry.



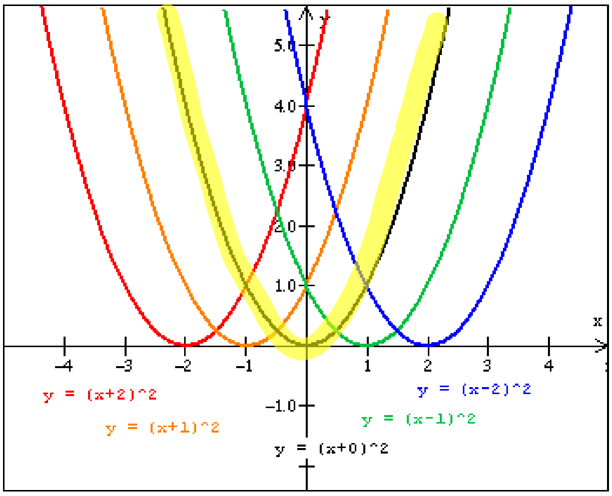
Graph of the Quadratic Function $f(x) = (x-h)^2$ (a horizontal shift):

Adding a constant to the x variable in $y = x^2$ before squaring has the effect of shifting the graph horizontally (left/right).

The graph of $y = (x-h)^2$ is the graph of $y = x^2$ shifted ***right*** by h units when h is a positive number.

The graph of $y = (x-h)^2$ is the graph of $y = x^2$ shifted ***left*** by h units when h is a negative number.

Notice that the vertex is at $(h, 0)$.
Notice that the graph still "opens up", and has a minimum value.
Notice that the vertical line $x = h$ is the new axis of symmetry.



Graph of the Quadratic Function $f(x) = ax^2$ (a reflection about the x-axis, and/or a vertical stretch):

Multiplying the x variable by a constant in $y = x^2$ after squaring can reflect the graph about the x-axis, or produce a vertical stretching of the graph.

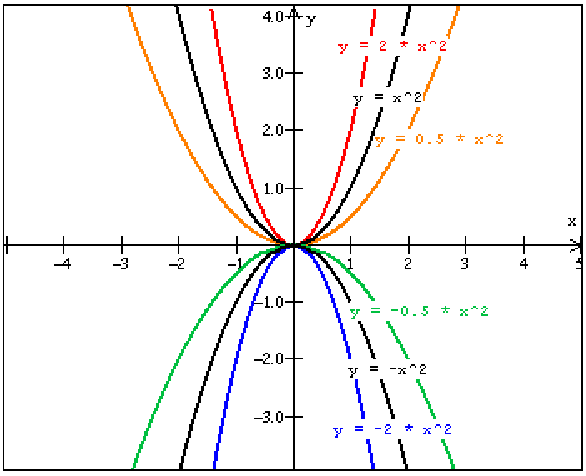
The graph of $y = ax^2$ is the graph of $y = x^2$ stretched vertically upwards when a is a positive number greater than 1.

The graph of $y = ax^2$ is the graph of $y = x^2$ compressed vertically upwards when a is a positive number less than 1.

The graph of $y = ax^2$ is the graph of $y = x^2$ reflected about the x-axis and stretched vertically downwards when a is a negative number less than -1.

The graph of $y = ax^2$ is the graph of $y = x^2$ reflected about the x-axis and compressed vertically downwards when a is a positive number greater than -1.

Notice that the vertex is at (0, 0).
Notice that the graph "opens up" when a>0 and "opens down" when a< 0.
Notice that the vertical line $x = 0$ is still the axis of symmetry.



Writing Quadratic Equations

General Form	Standard Form	Transformational Form
$y = ax^2 + bx + c$ where $a \neq 0$	$y = a(x - h)^2 + k$ where $a \neq 0$	$\frac{1}{a}(y - k) = (x - h)^2$
a = the vertical stretch (stretch factor) +/- = whether graph opens up or down or has a maximum or a minimum.	a = the vertical stretch (stretch factor) +/- = whether graph opens up or down or has a maximum or a minimum.	a = the vertical stretch (stretch factor) +/- = whether graph opens up or down or has a maximum or a minimum.
c = the value of the y-coordinate of the y-intercept.	**Not able to determine the y-intercept.**	**Not able to determine the y-intercept.**
Not able to determine vertex.	**(h, k)** gives us the coordinates of the vertex of the parabola.	**(h, k)** will give us the coordinates of the vertex of the parabola.
Not able to determine the equation of the axis of symmetry.	**x = h** gives us the equation of the axis of symmetry.	**x = h** gives us the equation of the axis of symmetry.

Range:

(opens up) $\{y \mid y \geq k, y \in \mathbb{R}\}$

(opens down) $\{y \mid y \leq k, y \in \mathbb{R}\}$

① $y = 1(x-3)^2 + 2$

$a = 1$
$h = 3$
$k = 2$

Stretch Factor: $a = 1$

opens: upward (minimum)

Vertex: $(3, 2)$

equation of the axis of symmetry: $x = 3$

Range: $\{y \mid y \geq 2, y \in \mathbb{R}\}$

Domain $\{x \mid x \in \mathbb{R}\}$

② $y = -3(x+2)^2 - 4$

$a = 3$
$h = -2$
$k = -4$

stretch: 3

opens: Downward

max/min: max $(-2, -4)$

vertex: $(-2, -4)$

axis of symm: $x = -2$

Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \leq -4, y \in \mathbb{R}\}$

Ex $\frac{1}{4}(y-2)=(x+3)^2$

$a=4 \quad h=-3 \quad k=2$

$-\frac{1}{2}y=(x-4)^2$

opens down

$-\frac{1}{2}(y-0)=(x-4)^2$

$a=2 \quad h=4 \quad k=0$

$\frac{1}{1}(y+3)=(x+4)^2$

$a=1 \quad h=-4 \quad k=-3$

Vertex	Stretch	axis of symmetry	Opens	Dom	Range
$(-4,-3)$	1	$x=-4$	up	$\{x \mid x \in R\}$	$\{y \mid y \geq -3, y \in R\}$

⑥ $4(y-1)=x^2$

$\frac{4}{1}(y-1)=(x-0)^2$

$a=\frac{1}{4} \quad h=0 \quad k=1$

NOTE:

Horizontal Shift

The ***h*** value in the standard/transformational form of the quadratic equation stands for the horizontal shift. This transformation means that you **shift EVERY point** to the left or right ***h*** units. Remember the following:

- If ***h*** is **positive**, you will shift the graph to the **right**.
- If ***h*** is **negative**, you will shift the graph to the **left**.

This can get a little tricky, as the standard/transformational form of the quadratic equation throws a wrench into things. The "***h***" is contained within $(x - h)^2$, therefore you will have to be very careful in your thinking!
If the sign in front of the "***h***" is negative, the graph will actually be shifted to the right and if the sign in front of the "***h***" is positive, the graph will actually be shifted to the left.

Vertical Shift

The vertical shift is represented by the variable ***k*** in the standard/transformational form of the quadratic equation. The following rules apply:

- If the value of ***k*** is **positive**, you shift EVERY point on the graph **UP *k*** units.
- If the value of ***k*** is **negative**, you shift EVERY point on the graph **DOWN *k*** units.

These rules are very easy to follow for the standard form of the quadratic equation. In transformational form, however, the "***k***" is contained within $(y - k)$. Therefore, if the sign in front of the "***k***" is negative, the graph will actually be shifted upward and if the sign in front of the "***k***" is positive, the graph will actually be shifted downward.

③ $y = x^2 + 1$

$y = 1(x + \underline{\underline{0}})^2 + 1$

$a = 1$

$h = 0$

$k = 1$