Warm Up

2. Factor each of the following:

a) $x^{27} - 1$ b) $(x^2+1)^{\frac{1}{2}} + 3(x^2+1)^{-\frac{1}{2}}$

a) $(x^9 - 1)(x^{18} + x^9 + 1)$

$(x^3 - 1)(x^6 + x^3 + 1)(x^{18} + x^9 + 1)$

$(x - 1)(x^2 + x + 1)(x^6 + x^3 + 1)(x^{18} + x^9 + 1)$

b) $(x^2+1)^{1/2} + 3(x^2+1)^{-1/2}$

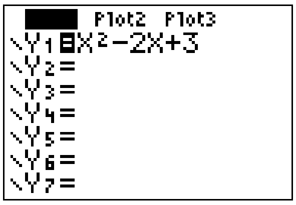
$(x^2+1)^{-1/2}\left[(x^2+1)^1 + 3\boxed{(x^2+1)^0}\right]$ $= 1$

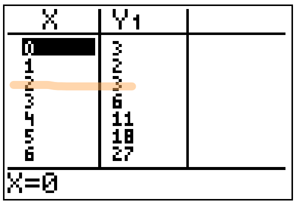
$(x^2+1)^{-1/2}(x^2+1+3)$

$(x^2+1)^{-1/2}(x^2+4)$ or $\dfrac{x^2+4}{(x^2+1)^{1/2}} = \dfrac{x^2+4}{\sqrt{x^2+1}}$

Limit of a Function

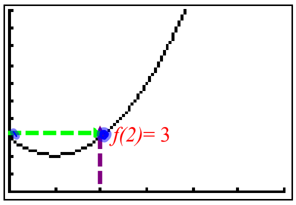
Let's examine the function $f(x) = x^2 - 2x + 3$



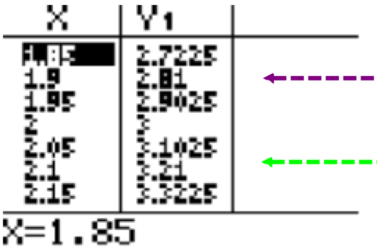


X	Y1
0	3
1	2
2	3
3	6
4	11
5	18
6	27

X=0



We can see that *f(2)*= 3...let's check the behaviour of *f* as we get closer and closer to $x = 2$.



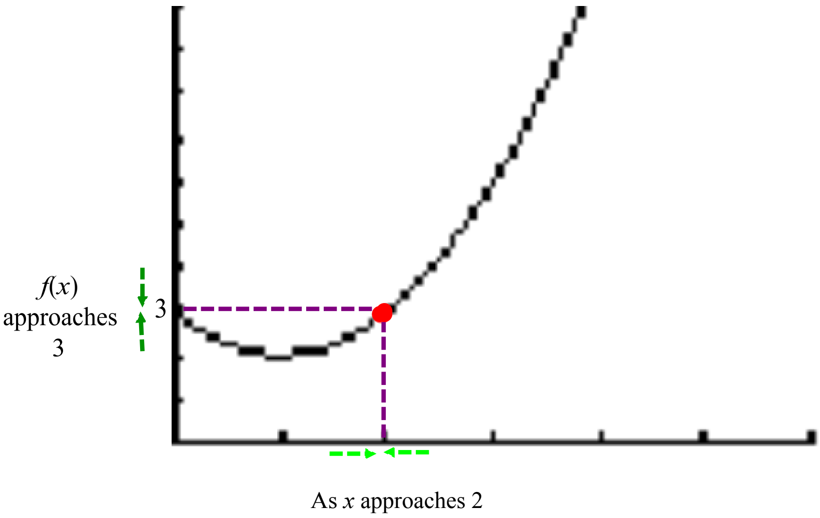
X	Y1
1.85	2.7225
1.9	2.81
1.95	2.9025
2	3
2.05	3.1025
2.1	3.21
2.15	3.3225

X=1.85

As x gets closer to 2 from the left
y is getting closer to 3.

As x gets closer to 2 from the right
y is getting closer to 3.

From the above, the notion of the limit of a function arises...



Notation: $\lim\limits_{x \to 2} f(x) = 3$ or $\lim\limits_{x \to 2} x^2 - 2x + 3 = 3$

"The limit of the function $f(x)$ as x approaches 2 is equal to 3."

The common sense definition of a limit...

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→ the intended height of a function.

A formal definition of a limit...

We write $\lim_{x \to a} f(x) = L$ if we can make the values of $f(x)$ arbitrarily close to L

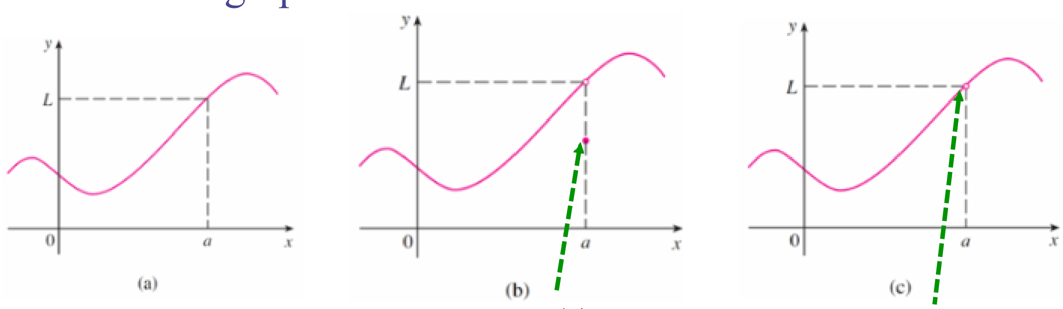
- (as close to L as we like)

by taking x to be sufficiently close to a

- (on either side of a)

but not equal to a.

Look at the graphs of these three functions...



Notice $f(a) \neq L$

Notice $f(a)$ is undefined

But in each case, regardless of what happens at a, it is true that

$$\lim_{x \to a} f(x) = L$$

Evaluating Limits

I. Using a Graph:

- We looked at this in the previous two examples

II. Algebraically:

- Direct Substitution...

Examples:

$$\lim_{x\to -2}\frac{x^2-2x+1}{x+3}$$

$$\lim_{x\to -2}\frac{(-2)^2-2(-2)+1}{(-2)+3}$$

$$\lim_{x\to -2}\frac{4+4+1}{1}=\boxed{9}$$

$$\lim_{x\to 3}\left(16-x^2\right)$$

$$\lim_{x\to 3} 16-(3)^2$$

$$\lim_{x\to 3} 16-9=\boxed{7}$$

* • Indeterminate limits... $\Rightarrow$ $\boxed{\text{Direct substitution leads to } \tfrac{0}{0}}$

- $\Rightarrow$ Factor
- $\Rightarrow$ Rationalize (look for radicals)
- $\Rightarrow$ Expand
- $\Rightarrow$ Find Common Denominators (look for fractions)

Examples:

$$\lim_{x\to 4}\frac{x^2-16}{x-4}$$

$$\lim_{x\to 4}\frac{(x+4)\cancel{(x-4)}}{\cancel{(x-4)}}=\boxed{8}$$

$$\lim_{h\to 0}\frac{\left(\sqrt{4+h}-2\right)}{h}\cdot\frac{\left(\sqrt{4+h}+2\right)}{\left(\sqrt{4+h}+2\right)}$$

$$\lim_{h\to 0}\frac{4+h-4}{h\left(\sqrt{4+h}+2\right)}$$

$$\lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}\left(\sqrt{4+h}+2\right)}=\boxed{\frac{1}{4}}$$

Try these...remember to use your algebra skills
to try and eliminate the **indeterminate form**.

$$\lim_{x\to 2}\frac{(x+2)^2-16}{x^2-4}$$

$$\lim_{x\to 2}\frac{(x+2+4)(x+2-4)}{(x+2)(x-2)}$$

$$\lim_{x\to 2}\frac{(x+6)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}=\frac{8}{4}=\boxed{2}$$

$$\lim_{x\to -2}\frac{x^4-16}{x^3+8}$$

$$\lim_{x\to -2}\frac{(x^2-4)(x^2+4)}{(x+2)(x^2-2x+4)}$$

$$\lim_{x\to -2}\frac{\cancel{(x+2)}(x-2)(x^2+4)}{\cancel{(x+2)}(x^2-2x+4)}$$

$$\lim_{x\to -2}\frac{(-4)(8)}{12}=\frac{-32}{12}=\boxed{\frac{-8}{3}}$$

$$\lim_{x\to 0}\frac{x^2+3x}{(x+2)^2-(x-2)^2}$$

$$\lim_{x\to 0}\frac{x(x+3)}{(x+2+(x-2))(x+2-(x-2))}$$

$$\lim_{x\to 0}\frac{x(x+3)}{2x(4)}$$

$$\lim_{x\to 0}\frac{\cancel{x}(x+3)}{8\cancel{x}}=\boxed{\frac{3}{8}}$$

$$\lim_{x\to 2}\frac{2x\cdot\frac{1}{x}-\frac{1}{2}\cdot 2x}{(x-2)\,2x}$$

$$\lim_{x\to 2}\frac{2-x}{2x(x-2)}$$

$$\lim_{x\to 2}\frac{-1\cancel{(x-2)}}{2x\cancel{(x-2)}}=\boxed{\frac{-1}{4}}$$

Homework