

Warm Up

Differentiate each of the following:

1. $f(x) = 6^{x^3} + \ln(\tan^{-1} 2x^4)$

$$f'(x) = 6^{x^3} (\ln 6) (3x^2) + \left(\frac{1}{\tan^{-1} 2x^4} \right) \left(\frac{8x^3}{1+(2x^4)^2} \right)$$

2. $y = (8x-1)^{\sqrt{x}}$

$$\ln y = \ln (8x-1)^{\sqrt{x}}$$

$$\ln y = [x^{1/2}] [\ln(8x-1)]$$

$$\cancel{y} \frac{y'}{y} = \left[x^{1/2} \left(\frac{8}{8x-1} \right) + \frac{1}{2} x^{-1/2} (\ln(8x-1)) \right] [(8x-1)^{\sqrt{x}}]$$

Derivative Rules

Exponential Functions

$$d(b^u) = b^u \cdot (\ln b) \cdot du, \text{ where } b \in R$$

$$d(e^u) = e^u \cdot du, \text{ base is Euler's number}$$

Logarithmic Functions

$$d(\log_b u) = \frac{1}{u \ln b} \cdot du, \text{ where } b \in R$$

$$d(\ln u) = \frac{1}{u} \cdot du, \text{ base is Euler's number}$$

Inverse Trigonometric Functions

$$\frac{d(\sin^{-1} u)}{du} = \frac{1}{\sqrt{1-u^2}} du \quad \frac{d(\csc^{-1} u)}{du} = \frac{-1}{u\sqrt{u^2-1}} du$$

$$\frac{d(\cos^{-1} u)}{du} = \frac{-1}{\sqrt{1-u^2}} du \quad \frac{d(\sec^{-1} u)}{du} = \frac{1}{u\sqrt{u^2-1}} du$$

$$\frac{d(\tan^{-1} u)}{du} = \frac{1}{1+u^2} du \quad \frac{d(\cot^{-1} u)}{du} = \frac{-1}{u^2+1} du$$

Quiz Monday: Derivatives of Transcendental Functions

- Inverse Trigonometric Functions
- Exponential Functions
- Logarithmic Functions

Practice Test

Solutions

May 06

1 a) $f(x) = \frac{1}{x^3 \ln 5} (3x^2) + e^{\sin 5x} (\cos 5x) (5)$ b) $y' = \frac{-1}{\sqrt{1-(2/3)^2}} (x^{-1/2}) - \frac{1}{\ln x^2} \left(\frac{1}{x^2} \right) (2x)$

c) $h'(t) = \frac{5^{\tan t} \ln 5 (\sec^2 t) \ln(3e^t + 5) - 5^{\tan t} \left(\frac{1}{3e^t + 5} \right) (e^t)}{[\ln(3e^t + 5)]^2}$

d) $\ln y = x \ln(5-2x^2)$
 $\frac{1}{y} y' = \left[\ln(5-2x^2) + x \left(\frac{1}{5-2x^2} \right) (-4x) \right] y$
 $y' = \left(\ln(5-2x^2) - \frac{4x^2}{5-2x^2} \right) (5-2x^2)^x$

e) $y' = \frac{1}{1 + [\ln^2(x^2-1)]^2} \left[3[\ln(x^2-1)]^2 \left(\frac{1}{x^2-1} \right) (5x^2) \right]$

f) $g'(x) = 4^{5x} \ln 4 (5) e^{\sin^2 \sqrt{x}} + 4^{5x} e^{\sin^2 \sqrt{x}} \left(\frac{1}{\sqrt{1-x}} \right) \left(\frac{1}{2} x^{-1/2} \right)$

2. $\ln y = 3 \ln(x^2-2x) + \ln 8x^5 - \frac{5}{2} \ln(5-x) - (x^2+2)$
 $\frac{1}{y} y' = \left[3 \left(\frac{2x-2}{x^2-2x} \right) + \frac{40x^4}{8x^5} - \frac{5}{2} \left(\frac{-1}{5-x} \right) - 5x^2 \right] y$
 $y' = \left(\frac{3(2x-2)}{x^2-2x} + \frac{5}{x} + \frac{5x}{5-x} - 5x^2 \right) \left[\frac{(x^2-2x)^3 (dx^2)}{(5-x)^{3/2} (e^{x^2+2})} \right]$

3. $e^{3x-y^5} (3-5y^4 \frac{dy}{dx}) = 5^{xy^3} \ln 5 (y^3 + x(3y^2 \frac{dy}{dx}))$
 $3e^{3x-y^5} - 5y^4 e^{3x-y^5} \frac{dy}{dx} = 5^{xy^3} \ln 5 y^3 + 3xy^2 5^{xy^3} \ln 5 \frac{dy}{dx}$
 $\frac{3e^{3x-y^5} - 5^{xy^3} \ln 5 y^3}{3xy^2 5^{xy^3} \ln 5 + 5y^4 e^{3x-y^5}} = \frac{dy}{dx}$

4. $\frac{dz}{dy} = \frac{1}{\sqrt{1-(y-3)^2}} (1) - 3y^{-2}$ with $x=0$

$\frac{dy}{dx} = 3e^x + 2x$

$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx} = \frac{dz}{dy} (3e^x + 2x)$

at $x=0 \dots y = 3+0 = 3$

$\frac{dz}{dy} = \frac{1}{\sqrt{1-0^2}} - 27$ $\frac{dy}{dx} = 3e^0 + 2(0) = 3$
 $= -26$

$\frac{dz}{dx} = -78$

5. $f'(x) = 2x e^{2x} + x^2 e^{2x} (2)$

$f'(x) = 2x e^{2x} (1+x)$

Critical Values: $x = -1, x = 0$

	$2x$	e^{2x}	$1+x$	f'/f	
$(-\infty, -1)$	-	+	-	+	Inc
$(-1, 0)$	-	+	+	-	Dec
$(0, \infty)$	+	+	+	+	Inc

Local Max: $(-1, \frac{1}{e^2})$

Local Min: $(0, 0)$

$$\textcircled{1} \text{e)} \quad y = \tan^{-1}(\ln^3(x^5-1))$$

$$y' = \left[\frac{1}{1 + (\ln^3(x^5-1))^2} \right] \left[3[\ln^2(x^5-1)] \left[\frac{5x^4}{x^5-1} \right] \right]$$

$$\textcircled{4} \quad z = \sin^{-1}(y-3) - y^3 \quad y = 3e^x + x^2$$

$$z = \sin^{-1}(3e^x + x^2 - 3) - (3e^x + x^2)^3$$

$$z' = \left[\frac{1}{\sqrt{1 - (3e^x + x^2 - 3)^2}} \right] [3e^x + 2x] - 3(3e^x + x^2)^2 (3e^x + 2x)$$

$$z' = \frac{3e^x + 2x}{\sqrt{1 - (3e^x + x^2 - 3)^2}} - 3(3e^x + x^2)^2 (3e^x + 2x)$$

$$z'(0) = \frac{3e^0 + 2(0)}{\sqrt{1 - (3e^0 + (0)^2 - 3)^2}} - 3(3e^0 + (0)^2)^2 (3e^0 + 2(0))$$

$$z'(0) = 3 - 3(3)^2(3)$$

$$z'(0) = 3 - 81$$

$$z'(0) = -78$$

$$\textcircled{5} f(x) = (x^2)e^{2x}$$

$$f'(x) = x^2(e^{2x})(2) + e^{2x}(2x)$$

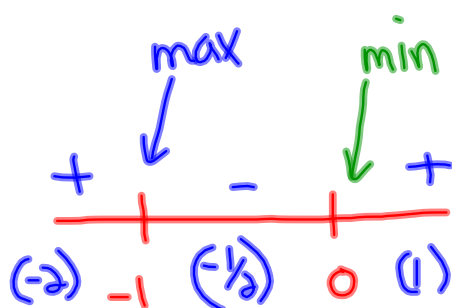
$$f'(x) = 2x^2e^{2x} + 2xe^{2x}$$

$$f'(x) = 2xe^{2x}[x+1]$$

cv:

$$0 = 2xe^{2x}[x+1]$$

$$\begin{array}{l|l} 2xe^{2x} = 0 & x+1 = 0 \\ x = 0 & x = -1 \end{array}$$



max ($x = -1$)

$$f(-1) = (-1)^2 e^{2(-1)} = 1e^{-2} = \frac{1}{e^2} \quad (-1, \frac{1}{e^2})$$

min ($x = 0$)

$$f(0) = (0)^2 e^{2(0)} = 0e^0 = 0(1) = 0 \quad (0, 0)$$

Attachments

Review of Transcendentals.doc

logs & arcfuctions test 2006.doc