Polar Coordinates

1. **Convert 4 - 3i to Polar form**

$a+bi \longrightarrow r\,cis\,\theta$

Find the radius *r*, using the Pythagorean relationship $r = \sqrt{x^2 + y^2}$

Find the related angle, α, using $\alpha = \tan^{-1}\left(\frac{|y|}{|x|}\right)$

Find the angle, θ, by determining the quadrant in which the terminal arm is located and using the related angle.

★

180-α	α
180+α	360-α

Remember from last semester

The polar form is ***rcis***θ

2. **Convert 2cis47° to rectangular form**

$r\,cis\,\theta \longrightarrow a+bi$

$$a = r\cos\theta$$

$$b = r\sin\theta$$

① $4-3i$

$\left.\begin{array}{l} a=4 \\ b=-3 \end{array}\right\}$ Quad 4

① $r=\sqrt{(4)^2+(-3)^2}$

$r=\sqrt{25}$

$r=5$

② $\alpha=\tan^{-1}\left(\frac{3}{4}\right)$

$\alpha=36.9°$

③ Quad 4

$\theta=360°-\alpha$

$\theta=360°-36.9$

$\theta=323.1°$

④ $\boxed{5cis323.1°}$

② $2cis47°$

$r=2$

$\theta=47°$

① $a=r\cos\theta$

$=2\cos47°$

$=1.36$

② $b=r\sin\theta$

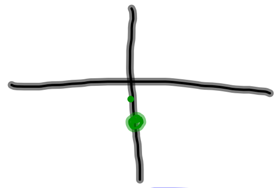
$=2\sin47°$

$=1.46$

③ $\boxed{1.36+1.46i}$

(19) c) $0-2i$

$a+bi \longrightarrow r\,\text{cis}\,\theta$



① $r=\sqrt{0^2+(-2)^2}$ ② $\alpha=\tan^{-1}\left(\frac{2}{0}\right)$ ③ $\boxed{2\,\text{cis}\,270°}$

$r=\sqrt{4}$ * $\alpha=270°$

$r=2$

(19) b) $-5\sqrt{3}+5i$

$a=-5\sqrt{3}$ Quad 2

$b=5$

① $r=\sqrt{(-5\sqrt{3})^2+(5)^2}$ ② $\alpha=\tan^{-1}\frac{5}{5\sqrt{3}}$ ③ $\theta=180°-30°$

$=\sqrt{75+25}$ $\theta=150°$

$\alpha=30°$

$=10$

④ $10\,\text{cis}\,150°$

$10\cos150°+10i\sin150°$

(21) b) (ii) $8(\cos225°+i\sin225°)$

$8\cos225°+8i\sin225°$ Polar form

$8\,\text{cis}\,225°$

$r=8$ $\theta=225°$

① $a=8\cos225$ ② $b=8\sin225$ ③ $\boxed{-5.65-5.65i}$

$=-5.65$ $=-5.65$

Operations with Complex Numbers in Polar Form

Multiply the following complex numbers

$(2+3i)(3+i) = 6+11i+3(i^2) = 3+11i$

$(\sqrt{13}\operatorname{cis}56.3^\circ)(\sqrt{10}\operatorname{cis}18.4^\circ) = (\sqrt{130}\operatorname{cis}74.7^\circ)$

Convert all complex numbers from rectangular form to Polar form.

① $r=\sqrt{(2)^2+(3)^2}$ ② $\alpha=\tan^{-1}\left(\frac{3}{2}\right)$ ③ Quad I ④ $\sqrt{13}\operatorname{cis}56.3^\circ$

$r=\sqrt{13}$

$\alpha=56.3^\circ$

$\theta=\alpha$

$\theta=56.3^\circ$

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① $r=\sqrt{(3)^2+(1)^2}$ ② $\alpha=\tan^{-1}\left(\frac{1}{3}\right)$ ③ Quad I ④ $\sqrt{10}\operatorname{cis}18.4$

$r=\sqrt{10}$

$\alpha=18.4^\circ$

$\theta=\alpha$

$\theta=18.4^\circ$

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① $r=\sqrt{(3)^2+(11)^2}$ ② $\alpha=\tan^{-1}\left(\frac{11}{3}\right)$ ③ Quad I ④ $\sqrt{130}\operatorname{cis}74$

$r=\sqrt{130}$

$\alpha=74.7^\circ$

$\theta=\alpha$

$\theta=74.7^\circ$

Do the same for the following complex numbers

$(1+4i)(3-2i)$

You may have noticed a shortcut when multiplying complex numbers in Polar form.

angles
θ

- When Multiplying, *multiply* the "r" values and *add* the arguments.
- When Dividing you *divide* the "r" values and *subtract* the arguments

Argument:

The angle from the positive real axis to the position vector representing a complex number in the complex plane. If the number is written in ***polar form*** as ***rcis***θ then θ is the argument and "r" is the modulus.

Examples

$(2cis150°)(3cis200°) = 6cis350°$

$(2\sqrt{2}cis60°)(3\sqrt{8}cis240°) = 6\sqrt{16}\,cis300° = 24cis300°$

$(3cis150°)(5cis240°) = 15cis390° = 15cis30°$

$* = 15\cos 30 + 15i\sin 30°$

$\dfrac{45cis120°}{3cis190°} = 15cis(-70°) = 15cis290°$

De Moivre's Theorem

* Complex #'s must be in polar form

$$(r cis\theta)^n = r^n cis n\theta$$

$$(\underline{1+i\sqrt{3}})^9$$

$a=1 \quad b=\sqrt{3}$

① $r=\sqrt{(1)^2+(\sqrt{3})^2}$ ② $\alpha=\tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$ ③ Quad I ④ $\underline{2cis60^\circ}$

$r=\sqrt{4}$ $\quad$ $\alpha=60^\circ$ $\quad$ $\theta=\alpha$

$r=2$ $\quad$ $\theta=60^\circ$

$$(2cis60^\circ)^9 = 2^9 cis(9\cdot 60) = 512cis540^\circ$$

Polar form $= \boxed{512cis\,180^\circ}$

If the question asked you to express your answer in rectangular form

① $a = 512\cos 180^\circ$ ② $b = 512\sin 180^\circ$ ③ $-512+0i$

$a=-512$ $\quad$ $b=0$ $\quad$ Rectangular form $\boxed{=-512}$

$512cis180^\circ$

$=512\cos180^\circ + 512i\sin180^\circ$

$=512(-1)+512i(0)$

$=\boxed{-512}$ Rectangular Form

Homework