Questions from Homework

① a) $\lim_{x\to 0} \dfrac{\frac{2}{x+2} - 1}{x}$ $\cdot (x+2)$

$\lim_{x\to 0} \dfrac{\frac{2}{x+2} - \frac{x+2}{x+2}}{x}$

$\lim_{x\to 0} \dfrac{2-x-2}{x(x+2)}$

$\lim_{x\to 0} \dfrac{-x}{x(x+2)} = \boxed{-\frac{1}{2}}$

$\lim_{x\to 0} \dfrac{\frac{2-x-2}{x+2}}{x}$

$\lim_{x\to 0} \dfrac{-x}{x+2} \cdot \dfrac{1}{x} = \boxed{-\frac{1}{2}}$

c) $\lim_{x\to 1} \dfrac{(x+2)^3 - 27}{x-1}$

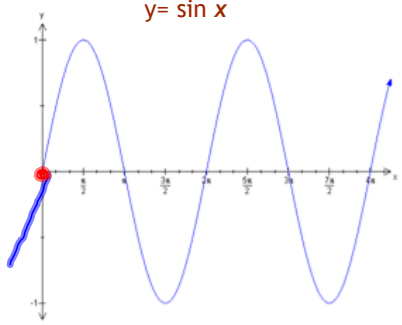
$\lim_{x\to 1} \dfrac{((x+2)-3)((x+2)^2+3(x+2)+9)}{(x-1)} = 9+9+9 = \boxed{27}$

d) $\lim_{x\to 7} \dfrac{(\sqrt{x+9}-4)(\sqrt{x+9}+4)}{(x-7)(\sqrt{x+9}+4)}$

$\lim_{x\to 7} \dfrac{x+9-16}{(x-7)(\sqrt{x+9}+4)} = \dfrac{1}{4+4} = \boxed{\frac{1}{8}}$

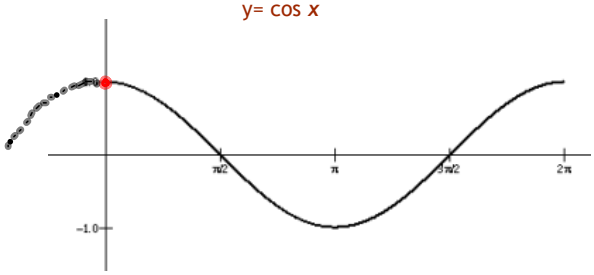
Limits of Trigonometric Functions

y= sin *x*



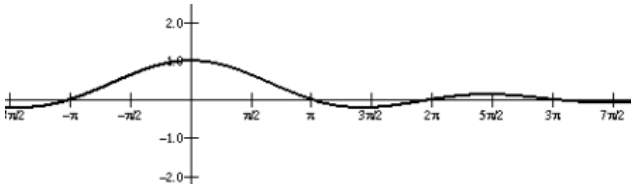
$$\lim_{x\to 0} \sin x = 0$$

y= cos *x*



$$\lim_{x\to 0} \cos x = 1$$

Here is the graph of $y = \dfrac{\sin x}{x}$



X	Y1
-3	.04704
-2	.45465
-1	.84147
0	ERROR
1	.84147
2	.45465
3	.04704

X= -3

Examine the following limit... $\lim_{x\to 0} \dfrac{\sin x}{x}$

Identity

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{x}{\sin x} = 1$$

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$$

$$\lim_{\theta\to 0} \frac{\theta}{\sin\theta} = 1$$

Examples: $\lim_{x\to 0}\frac{x}{\sin x}=1$ $\lim_{x\to 0}\frac{\sin x}{x}=1$

$$\lim_{x\to 0}\left(\frac{\sin 5x}{5x}\right)(5)$$

$$\lim_{x\to 0}(1)(5)=\boxed{5}$$

$$\lim_{x\to 0}\frac{8x}{\sin 5x}$$

$$\lim_{x\to 0}\left(\frac{5x}{\sin 5x}\right)\frac{8}{5}$$

$$\lim_{x\to 0}(1)\left(\frac{8}{5}\right)=\boxed{\frac{8}{5}}$$

$$\lim_{x\to \frac{\pi}{4}}\frac{4x}{\sin x}$$ ← Direct Sub →

$$\lim_{x\to 0}\frac{6x}{\cos 3x}$$

$$\lim_{x\to \frac{\pi}{4}}\frac{4\left(\frac{\pi}{4}\right)}{\sin\left(\frac{\pi}{4}\right)}=\frac{\pi}{\frac{1}{\sqrt{2}}}=\boxed{\pi\sqrt{2}}$$

$$\lim_{x\to 0}\frac{6(0)}{\cos 3(0)}=\frac{0}{1}=\boxed{0}$$

$$\lim_{x\to 0}\frac{\sin 2x}{\tan 7x}$$

$$\lim_{x\to 0}\frac{\sin 2x}{\frac{\sin 7x}{\cos 7x}}$$

$$\lim_{x\to 0}\sin 2x\left(\frac{\cos 7x}{\sin 7x}\right)$$

$$\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)\left(\frac{7x}{\sin 7x}\right)\cos 7x\left(\frac{2x}{7x}\right)$$

$$\lim_{x\to 0}(1)(1)(1)\left(\frac{2}{7}\right)=\boxed{\frac{2}{7}}$$

$$\lim_{x\to 0}\frac{\sin^3 2x}{5x^3+10x^4}$$

$$\lim_{x\to 0}\frac{\sin^3 2x}{5x^3(1+2x)}$$

$$\lim_{x\to 0}\left(\frac{\sin^3 2x}{x^3}\right)\left(\frac{1}{5(1+2x)}\right)$$

$$\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)^3\left(\frac{1}{5(1+2x)}\right)(8)$$

$$\lim_{x\to 0}(1)^3\left(\frac{1}{5(1)}\right)(8)=\boxed{\frac{8}{5}}$$

Homework

Page 306 & 307
#7, 9, 15, 16, 18, 20, 22, 23, 26, 27, 31, 37

$$\lim_{x\to 0} \frac{(\cos x - 1)(\cos x + 1)}{x^2(\cos x + 1)}$$

Hint: multiply by $(\cos x + 1)$

$$\lim_{x\to 0} \frac{\cos^2 x - 1}{x^2(\cos x + 1)}$$

$$\lim_{x\to 0} \frac{-\sin^2 x}{x^2(\cos x + 1)}$$

$$\lim_{x\to 0} -\left(\frac{\sin x}{x}\right)^2\left(\frac{1}{\cos x + 1}\right)$$

$$\lim_{x\to 0} -(1)^2\left(\frac{1}{2}\right) = \boxed{\frac{-1}{2}}$$