9. Graph each system. Determine a solution for each.

a) $\{(x, y) \mid y \geq 0.5x, x \in \mathbb{R}, y \in \mathbb{R}\}$

$\{(x, y) \mid x + y < 7, x \in \mathbb{R}, y \in \mathbb{R}\}$

Equations of the boundaries:

→ $y = 0.5x$ → $x + y = 7$

2 points on each boundary:

→ $y = 0.5x$ (x-int & y-int will both be 0) → $x + y = 7$

If $x = 2$:

$y = 0.5(2)$ $(2, 1)$

$y = 1$

x-int:

$x + 0 = 7$

$x = 7$

y-int:

$0 + y = 7$

$y = 7$

Test Points:

→ $y \geq 0.5x$; $(6, 0)$

L.S	R.S.
y	$0.5x$
0	$0.5(6)$
0	3

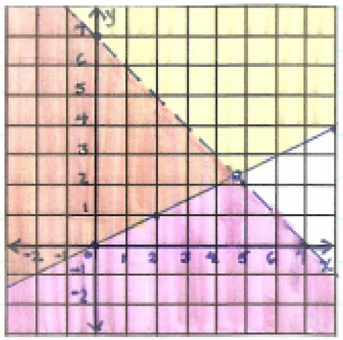
Since 0 is not ≥ 3, (0,0) is not located in solution region.

→ $x + y < 7$; $(0, 0)$

L.S	R.S
$x+y$	7
$0+0$	
0	

Since $0 < 7$, (0,0) is located in the solution region.

GRAPH:



Solution:

For example: (2, 4)

b) $\{(x, y) \mid y - 2x > 2, x \in W, y \in W\}$

$\{(x, y) \mid x + 2y < 12, x \in W, y \in W\}$

Equations of the boundaries:

→ $y - 2x = 2$ → $x + 2y = 12$

2 points on each boundary:

→ $y - 2x = 2$

x-int:
$0 - 2x = 2$
$\frac{-\cancel{2}x}{-\cancel{2}} = \frac{2}{-2}$
$x = -1$

y-int:
$y - 2(0) = 2$
$y - 0 = 2$
$y = 2$

→ $x + 2y = 12$

x-int:
$x + 2(0) = 12$
$x + 0 = 12$
$x = 12$

y-int:
$0 + 2y = 12$
$\frac{\cancel{2}y}{\cancel{2}} = \frac{12}{2}$
$y = 6$

Test Points:

→ $y - 2x > 2$; $(0,0)$

L.S	R.S
$y - 2x$	2
$0 - 2(0)$	
$0 - 0$	
0	

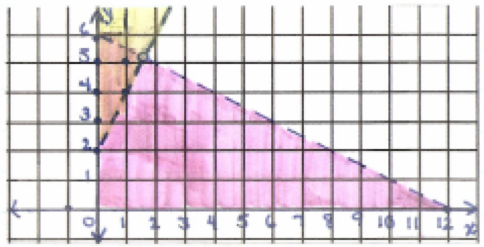
Since 0 is not > 2, (0,0) is not located in the solution region

→ $x + 2y < 12$

L.S.	R.S.
$x + 2y$	12
$0 + 2(0)$	
$0 + 0$	
0	

Since 0<12, (0,0) is located in the solution region.

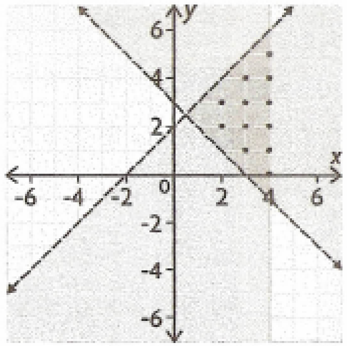
GRAPH:



Solution:

For example: (1,5)

10. The graph of a system of linear inequalities is shown, where the objective function is $P = 1.5x + 4y$.



a) Determine the vertices of the feasible region.

(0.5,2.5) , (4,-1) , (4,6)

b) What is the minimum solution for the system? (4,-1)

c) If P represents the amount of profit, in thousands of dollars, what is the minimum profit that can be made? $2,000

d) What is the maximum solution for the system? (4, 6)

e) If P represents the amount of profit, in thousands of dollars, what is the maximum profit that can be made? $30 000

* For (0.5,2.5): $P = 1.5(0.5) + 4(2.5)$
$P = 0.75 + 10$
$P = 10.75$

For (4,-1): $P = 1.5(4) + 4(-1)$
$P = 6 - 4$
$P = 2$

For (4,6): $P = 1.5(4) + 4(6)$
$P = 6 + 24$
$P = 30$

11. A snack machine sells granola bars and bags of trail mix.

- The machine holds, at most, 200 units of snacks.
- At least 4 granola bars are sold for each bag of trail mix.
- Each granola bar sells for $1.00, and each bag of trail mix sells for $1.25.

Let g represent the number of granola bars and t represent the number of bags of trail mix.

a) Write a linear inequality to represent the number of units of snacks the machine holds.

$\{(g, t) \mid g + t \leq 200, g \in W, t \in W\}$

b) Write a linear inequality to represent the number of granola bars sold compared to bags of trail mix.

$\{(g, t) \mid g \geq 4t, g \in W, t \in W\}$

g	t
4	1
8	2
12	3

c) Write an objective function for the revenue, R, from snack sales.

$R = 1.00g + 1.25t$

WRITTEN RESPONSE

12. Lite Lights manufactures two types of book light: type A is a solar-powered light; type B requires batteries. In one day, the company can make at most 55 of type A and 65 of type B. Type A requires 4 h to produce, and type B requires 2 h to produce. The production team can work a total of 240 hours each day.

a) Define the variables for this situation. State any restrictions.

Let a represent the number of type A book lights produced in one day.

Let b represent the number of type B book lights produced in one day.

Restrictions: $a \in W$, $b \in W$.

b) Write a system of linear inequalities to model this situation.

$a \geq 0$, $b \geq 0$, $a \leq 55$, $b \leq 65$, $4a + 2b \leq 240$

c) Graph the system of linear inequalities on the grid provided.

Equations of the boundaries:

→ $a = 55$ → $b = 65$ → $4a + 2b = 240$

2 points on each boundary:

→ $a = 55$ (O.K) → $b = 65$ (O.K) → $4a + 2b = 240$

* Vertical Line *Horizontal line

a-int:

$4a + 2(0) = 240$

$\frac{\cancel{4}a}{\cancel{4}} = \frac{240}{4}$

$a = 60$

b-int:

$4(0) + 2b = 240$

$\frac{\cancel{2}b}{\cancel{2}} = \frac{240}{2}$

$b = 120$

Test Points:

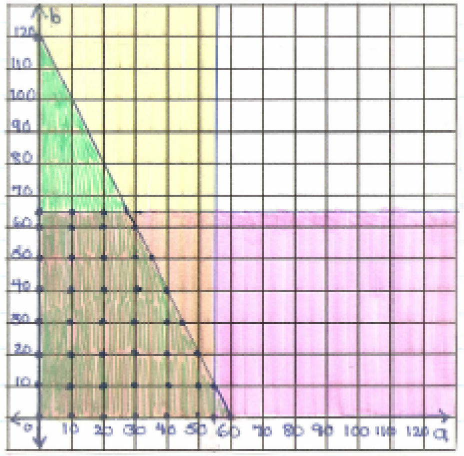
→ $a \le 55$ (O.K.) → $b \le 65$ (O.K) → $4a + 2b \le 240$; (0,0)

* Shade on left of line

* Shade below the line

L.S.	R.S.
$4a+2b$	240
$4(0)+2(0)$	
$0+0$	*Since $0 \le 240$, (0,0) is located in the solution region.
0	

GRAPH:



d) Determine the vertices of the feasible region.

(0,0) (0,65) (27.5,65) (55,10) (55,0)

Approx.

e) Which of the following are solutions to the system?

(55, 65), (25, 25), (45, 50)

What does each solution mean?

(55,65) and (45,50) are not solutions. (not located in the feasible region).

(25,25) is a solution. The company can make 25 type A lights and 25 type B lights.

13. Jenna and Rhiana sell tacos and burritos from a food cart.

- No more than 50 tacos and 75 burritos can be made each day.
- Jenna and Rhiana can make no more than 110 items, in total, each day.
- It costs \$0.75 to make a taco and \$1.25 to make a burrito.

Create an optimization model and use it to determine the maximum and minimum costs to produce the food items.

Let t represent the number of tacos that can be made in a day.
Let b represent the number of burritos that can be made in a day.
Let C represent the cost of making the goods.

Restrictions: $t \in W$, $b \in W$

Constraints: $t \geq 0$, $b \geq 0$, $t \leq 50$, $b \leq 75$, $t + b \leq 125$.

Objective Function: $C = 0.75t + 1.25b$

Equations of the boundaries:

→ $t = 50$ → $b = 75$ → $t + b = 125$

2 points on each boundary (x-int & y-int):

→ $t = 50$ (O.K) → $b = 75$ (O.K.) → $t + b = 125$

* Vertical Line * Horizontal Line

t-int: $t + 0 = 125$, $t = 125$

b-int: $0 + b = 125$, $b = 125$

Test Points:

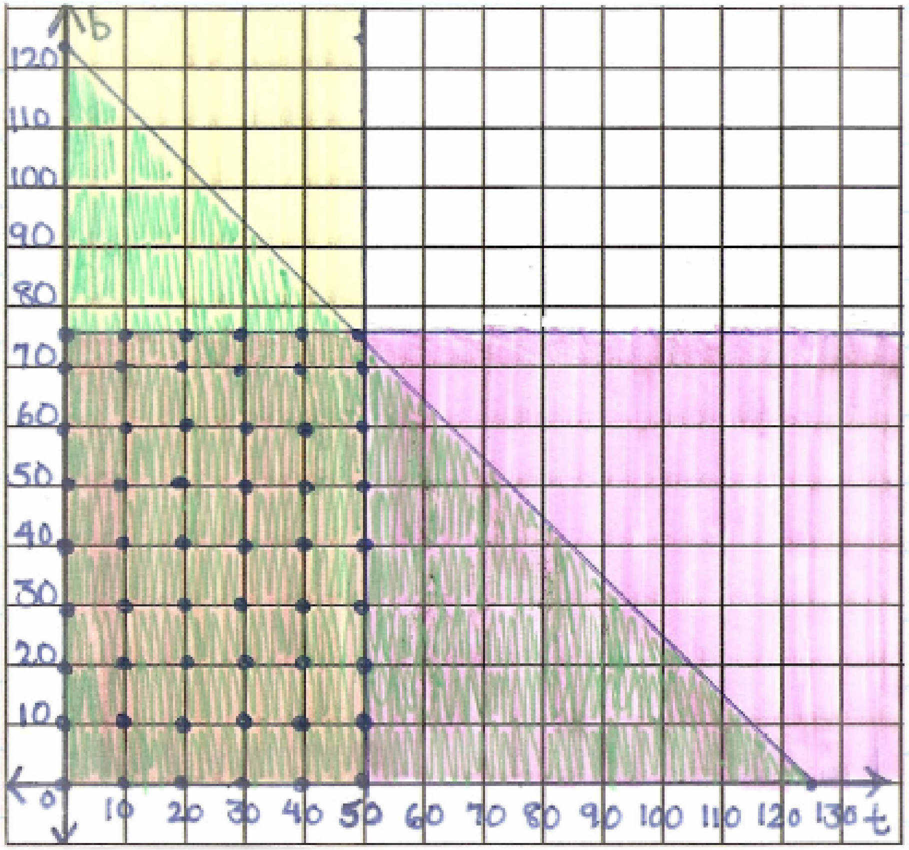
→ $t \leq 50$ (O.K.) → $b \leq 75$ → $t + b \leq 125$; (0,0)

* Shaded to the left of the line * Shaded below the line.

L.S.	R.S
$t + b$	125
$0 + 0$	
0	

Since $0 \leq 125$, (0,0) is located in the solution region.

GRAPH:



Vertices of feasible region:

(0,0), (0,75), (50,75) and (50,0)

For (0,0): $C = 0.75t + 1.25b$
$C = 0.75(0) + 1.25(0)$
$C = \$0$

For (0,75): $C = 0.75t + 1.25b$
$C = 0.75(0) + 1.25(75)$
$C = 0 + 93.75$
$C = \$93.75$

For (50,75): $C = 0.75t + 1.25b$
$C = 0.75(50) + 1.25(75)$
$C = 37.50 + 93.75$
$C = \$131.25$

For (50,0): $C = 0.75t + 1.25b$
$C = 0.75(50) + 1.25(0)$
$C = 37.50 + 0$
$C = \$37.50$.

* Minimum Cost => \$0 (0 tacos / 0 burritos)
Maximum Cost => \$131.25 (50 tacos / 75 burritos)