Chain Rule:

④ $G(x) = \sqrt{x^4 - x + 1} = (x^4 - x + 1)^{1/2}$

$G'(x) = \frac{1}{2}(x^4 - x + 1)^{-1/2}(4x^3 - 1)$

$G'(x) = \frac{1}{2(x^4 - x + 1)^{1/2}} \cdot (4x^3 - 1)$

$G'(x) = \frac{4x^3 - 1}{2\sqrt{x^4 - x + 1}}$

⑧ $y = \frac{4}{\sqrt{9 - x^2}} = \frac{4}{(9 - x^2)^{1/2}} = 4(9 - x^2)^{-1/2}$

$y' = -2(9 - x^2)^{-3/2}(-2x)$

$y' = 4x(9 - x^2)^{-3/2}$

$y' = \frac{4x}{(9 - x^2)^{3/2}} = \frac{4x}{\sqrt{(9 - x^2)^3}}$

⑩ $y = \sqrt{x + \sqrt{x}} = (x + \sqrt{x})^{1/2} = (x + x^{1/2})^{1/2}$

$y' = \frac{1}{2}(x + x^{1/2})^{-1/2}\left(1 + \frac{1}{2}x^{-1/2}\right)$

$y' = \left[\frac{1}{2(x + \sqrt{x})^{1/2}}\right]\left[\frac{1}{1} + \frac{1}{2\sqrt{x}}\right]$ Add by finding common denom

$y' = \left[\frac{1}{2\sqrt{x + \sqrt{x}}}\right]\left[\frac{2\sqrt{x} + 1}{2\sqrt{x}}\right]$

$y' = \frac{2\sqrt{x} + 1}{4\sqrt{x}\sqrt{x + \sqrt{x}}}$ or $\frac{2\sqrt{x} + 1}{4\sqrt{x^2 + x^{3/2}}}$

Limits

① a) $$\lim_{x\to 0} \frac{\cancel{(x+2)}\,\frac{2}{\cancel{x+2}} - \frac{1}{1}\,(x+2)}{x\,(x+2)}$$

$$\lim_{x\to 0} \frac{2-(x+2)}{x(x+2)}$$

$$\lim_{x\to 0} \frac{\cancel{2}-x-\cancel{2}}{x(x+2)}$$

$$\lim_{x\to 0} \frac{-\cancel{x}^{\,1}}{{}^{1}\cancel{x}(x+2)} = \boxed{-\frac{1}{2}}$$

b) $$\lim_{x\to \infty} \frac{(2-3x^2)^2}{6x^4-7x^2-5}$$

$$\lim_{x\to \infty} \frac{4-12x^2\boxed{+9}x^4}{\boxed{6}x^4-7x^2-5} = \frac{9}{6} = \boxed{\frac{3}{2}}$$

Review:

④ $g(x) = (x^2-3x+4)(2x^2+4x)$

$g'(x) = (x^2-3x+4)(4x+4) + (2x-3)(2x^2+4x)$

$g'(x) = 4x^3+4x^2-12x^2-12x+16x+16 + 4x^3+8x^2-6x^2-12x$

$g'(x) = 8x^3-6x^2-8x+16$

Differentiation Rules:

⑥ $y = (1+x)^{10}$ $x = \underline{\underline{0}}$ $y = \underline{\underline{1}}$ $(0,1)$

* $y = (1+0)^{10} = \underline{\underline{1}}$

① Find Derivative

$y' = 10(1+x)^9(1)$

$y' = 10(1+x)^9$

② Find Slope $(x=0)$

$y'(0) = 10(1+0)^9$

$= 10(1)^9$

$\boxed{= 10}$

m

③ Equation

$y - y_1 = m(x - x_1)$

$y - 1 = 10(x - 0)$

$y - 1 = 10x$

$\boxed{0 = 10x - y + 1}$

③ Sum & Difference Rule:

a) $y = x^{10} + 20x^5 - 12\sqrt[4]{x^3} + 30$

$= x^{10} + 20x^5 - 12x^{3/4} + 30$

$y' = 10x^9 + 100x^4 - 9x^{-1/4} + 0$

$= 10x^9 + 100x^4 - \dfrac{9}{x^{1/4}}$

⑧ $y = \frac{4}{\sqrt{9-x^2}} = \frac{4}{(9-x^2)^{1/2}} = 4(9-x^2)^{-1/2}$

$y' = -2(9-x^2)^{-3/2}(-2x)$

$= 4x(9-x^2)^{-3/2}$

$= \frac{4x}{(9-x^2)^{3/2}}$ or $\frac{4x}{\sqrt{(9-x^2)^3}}$

Limits

① a) $$\lim_{x\to0} \frac{\cancel{(x+2)}\,\frac{2}{\cancel{x+2}} - 1\,(x+2)}{x\,(x+2)}$$

$$\lim_{x\to0} \frac{2-x-2}{x(x+2)}$$

$$\lim_{x\to0} \frac{-\cancel{x}}{\cancel{x}(x+2)} = \frac{-1}{2}$$

Piecewise Functions:

$>, <$ → open dot

$\geq, \leq, =$ → closed dot

④ $$\boxed{f'(x) = \lim_{h\to 0} \frac{\boxed{f(x+h)} - \boxed{f(x)}}{h}}$$

b) $f(x) = \boxed{\dfrac{2x-1}{4x}}$ | $f(x+h) = \dfrac{2(x+h)-1}{4(x+h)}$

$$= \boxed{\frac{2x+2h-1}{4x+4h}}$$

$$f'(x) = \lim_{h\to 0} \frac{\frac{2x+2h-1}{(4x+4h)} - \left(\frac{2x-1}{(4x)}\right)}{h}$$

$$= \lim_{h\to 0} \frac{4x(2x+2h-1) - (2x-1)(4x+4h)}{h(4x)(4x+4h)}$$

$$= \lim_{h\to 0} \frac{8x^2+8xh-4x-(8x^2+8xh-4x-4h)}{h(4x)(4x+4h)}$$

$$= \lim_{h\to 0} \frac{\cancel{8x^2}+\cancel{8xh}\cancel{-4x}\cancel{-8x^2}\cancel{-8xh}\cancel{+4x}+4h}{h(4x)(4x+4h)}$$

$$= \lim_{h\to 0} \frac{4\cancel{h}}{\cancel{h}(4x)(4x+4h)} = \frac{4}{(4x)^2} = \frac{4}{16x^2} = \boxed{\frac{1}{4x^2}}$$

Chain Rule:

(18) $y = \sqrt{x+\sqrt{x}} = (x+\sqrt{x})^{1/2} = (x+x^{1/2})^{1/2}$

$$y' = \frac{1}{2}(x+x^{1/2})^{-1/2}\left(1+\frac{1}{2}x^{-1/2}\right)$$

$$y' = \left[\frac{1}{2(x+x^{1/2})^{1/2}}\right]\left[\frac{1}{1}+\frac{1}{2x^{1/2}}\right]$$

$$y' = \left[\frac{1}{2\sqrt{x+\sqrt{x}}}\right]\left[\frac{2\sqrt{x}+1}{2\sqrt{x}}\right]$$

$$y' = \frac{2\sqrt{x}+1}{4\sqrt{x}\sqrt{x+\sqrt{x}}} \quad \text{or} \quad \frac{2x^{1/2}+1}{4x^{1/2}(x+x^{1/2})^{1/2}}$$

$$\lim_{x \to \infty} \frac{1-9x^2}{3x^2+2x+1} = \frac{-9}{3} = \boxed{-3}$$

$$\lim_{x \to \infty} \frac{1-9x^2}{3x+1} = \boxed{DNE}$$

$$\lim_{x \to \infty} \frac{3x+1}{1-9x^2} = \boxed{0}$$

Ex 2.5

4

$f(2) = \boxed{3}$

$f'(2) = \boxed{5}$

$g(2) = \boxed{-1}$

$g'(2) = \boxed{-4}$

$$\left(\frac{f}{g}\right)'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2}$$

$$= \frac{(-1)(5) - (3)(-4)}{(-1)^2}$$

$$= \frac{-5 + 12}{1}$$

$$= 7$$

Ex: 2.5

⑥ $y = \frac{x^2}{2x+5}$

$y' = \frac{(2x+5)(2x) - x^2(2)}{(2x+5)^2}$

$y' = \frac{4x^2 + 10x - 2x^2}{(2x+5)^2}$

$y' = \frac{2x^2 + 10x}{(2x+5)^2} = \frac{2x(x+5)}{(2x+5)^2}$

$\frac{0}{1} = \frac{2x(x+5)}{(2x+5)^2}$

$(2x)(x+5) = 0$

$2x = 0$	$x+5=0$
$x = 0$	$x = -5$

$x = 0$	$x = -5$
$y = \frac{(0)^2}{2(0)+5}$	$y = \frac{(-5)^2}{2(-5)+5}$
$= \frac{0}{5}$	$= \frac{25}{-5}$
$= 0$	$= -5$
$(0,0)$	$(-5,-5)$