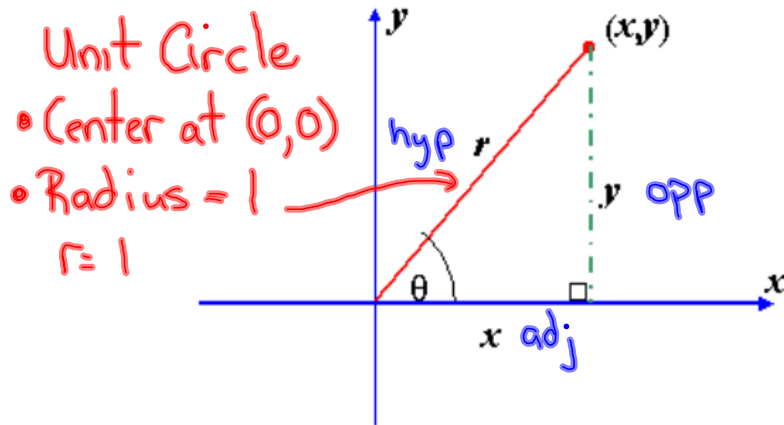


Trig Identities

Reciprocal Identities



$$\sin \theta = \frac{y}{r} = \frac{y}{1} = y \quad \cos \theta = \frac{x}{r} = \frac{x}{1} = x \quad \tan \theta = \frac{y}{x}$$

$$\csc \theta = \frac{r}{y} = \frac{1}{y} \quad \sec \theta = \frac{r}{x} = \frac{1}{x} \quad \cot \theta = \frac{x}{y}$$

Reciprocal Identities

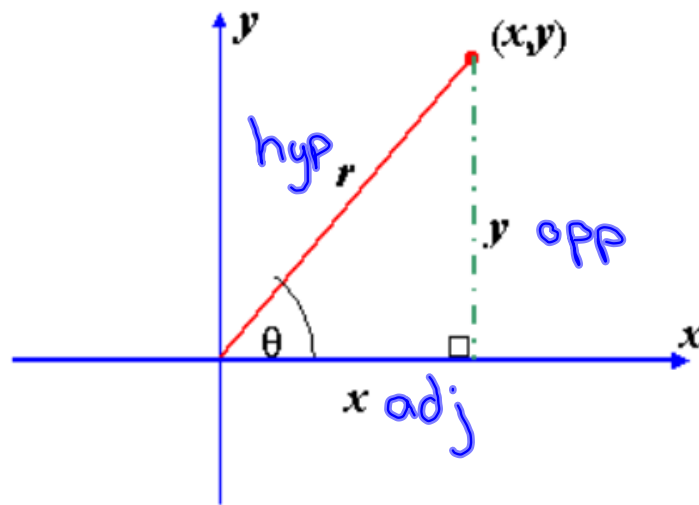
$$\sin \theta = \frac{1}{\csc \theta} \quad \cos \theta = \frac{1}{\sec \theta} \quad \tan \theta = \frac{1}{\cot \theta}$$

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

$$\sin^2 \theta = \frac{1}{\csc^2 \theta} \quad \cos^2 \theta = \frac{1}{\sec^2 \theta} \quad \tan^2 \theta = \frac{1}{\cot^2 \theta}$$

$$\csc^2 \theta = \frac{1}{\sin^2 \theta} \quad \sec^2 \theta = \frac{1}{\cos^2 \theta} \quad \cot^2 \theta = \frac{1}{\tan^2 \theta}$$

Quotient Identities



$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{x} = \frac{\sin \theta}{\cos \theta}$$

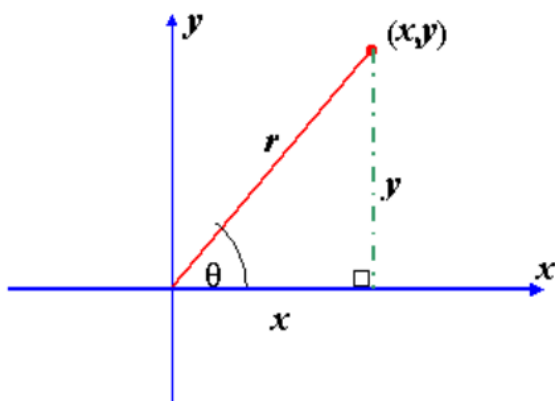
$$\cot \theta = \frac{\text{adj}}{\text{opp}} = \frac{x}{y} = \frac{\cos \theta}{\sin \theta}$$

Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$
$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\tan^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta}$$
$$\cot^2 \theta = \frac{\cos^2 \theta}{\sin^2 \theta}$$

Pythagorean Identities



$$\boxed{r=1}$$

$$\div r^2$$

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2}$$

$$x^2 + y^2 = 1$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\boxed{\sin^2 \theta + \cos^2 \theta = 1}$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\div x^2$$

$$\frac{x^2}{x^2} + \frac{y^2}{x^2} = \frac{r^2}{x^2}$$

$$\frac{\cos^2 \theta + \frac{\sin^2 \theta}{\cos^2 \theta}}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\boxed{\tan^2 \theta + 1 = \sec^2 \theta}$$

$$\tan^2 \theta = \sec^2 \theta - 1$$

$$1 = \sec^2 \theta - \tan^2 \theta$$

$$\div y^2$$

$$\frac{x^2}{y^2} + \frac{y^2}{y^2} = \frac{r^2}{y^2}$$

$$\frac{\frac{\cos^2 \theta}{\sin^2 \theta} + \frac{\sin^2 \theta}{\sin^2 \theta}}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

$$\boxed{\cot^2 \theta + 1 = \csc^2 \theta}$$

$$\cot^2 \theta = \csc^2 \theta - 1$$

$$1 = \csc^2 \theta - \cot^2 \theta$$

Trigonometric Identities

You must know these!

Quotient

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Reciprocal

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

Strategies for Proving Identities:

- Work on the most complex side and simplify so it has the same form as the simpler side
- Methods used in simplifying: direct substitution, factoring, finding a common denominator, multiplying by the conjugate → $1 + \cos\theta \rightarrow 1 - \cos\theta$

* $\sin\theta + \cos\theta = \text{good guys}$.

Prove the following:

$$\frac{\tan x}{\sin x} = \sec x$$
$$\frac{\frac{\sin x}{\cos x}}{\sin x}$$
$$\frac{\cancel{\sin x}}{\cos x} \cdot \frac{1}{\cancel{\sin x}}$$
$$\frac{1}{\cos x}$$
$$\frac{1}{\cos x}$$

$$\cos \theta \cdot \sec \theta = 1$$

$$\cancel{\cos \theta} \cdot \frac{1}{\cancel{\cos \theta}}$$

$$1$$

Prove the following:

$$\cot \theta \cdot \sin \theta = \cos \theta$$

$$\frac{\cos \theta}{\cancel{\sin \theta}} \cdot \cancel{\sin \theta}$$

$$\cos \theta$$

$$\frac{\cos x}{\tan x} = \frac{1 - \sin^2 x}{\sin x}$$

$$\frac{\cos x}{\frac{\sin x}{\cos x}}$$

$$\cos x \cdot \frac{\cos x}{\sin x}$$

$$\frac{\cos^2 x}{\sin x}$$

$$\frac{\cos^2 x}{\sin x}$$

Factor 

Ex. Prove that $\sin y + \sin y \cot^2 y = \csc y$

$$\sin y (\underline{1 + \cot^2 y})$$

$$\sin y (\csc^2 y)$$

$$\cancel{\sin y} \left(\frac{1}{\cancel{\sin y}} \right)$$

$$\boxed{\frac{1}{\sin y}}$$

$$\boxed{\frac{1}{\sin y}}$$

Homework

1. $\tan \theta \cos \theta = \sin \theta$

2. $\cot \theta \sec \theta = \csc \theta$

3. $\frac{1 + \cot^2 \theta}{\csc^2 \theta} = 1$

4. $\frac{\tan^2 \theta}{1 + \tan^2 \theta} = \sin^2 \theta$

5. $\frac{\tan^2 \theta}{\sin^2 \theta} = 1 + \tan^2 \theta$