

Warm Up

Prove the following identity:

$$\frac{\sin x}{1 - \cos x} - \frac{\sin x \cos x}{1 + \cos x} = \boxed{\csc x} (1 + \cos^2 x)$$

$$\frac{\sin x (1 + \cos x) - \sin x \cos x (1 - \cos x)}{\text{FOIL} \rightarrow (1 - \cos x)(1 + \cos x)}$$

$$\frac{\sin x + \cancel{\sin x \cos x} - \cancel{\sin x \cos x} + \sin x \cos^2 x}{\boxed{1 - \cos^2 x}}$$

$$\text{Factor} \rightarrow \frac{\sin x + \sin x \cos^2 x}{\sin^2 x}$$

$$\frac{\cancel{\sin x} (1 + \cos^2 x)}{\cancel{\sin x}}$$

$$\boxed{\frac{1 + \cos^2 x}{\sin x}}$$

$$\frac{1 + \cos^2 x}{\sin x}$$

$$\boxed{\frac{1 + \cos^2 x}{\sin x}}$$

Questions from Homework

$$\textcircled{8} (\sin x - \cos x)^2 + (\sin x + \cos x)^2 = \boxed{2}$$

$$\sin^2 x - 2\sin x \cos x + \cos^2 x + \sin^2 x + 2\sin x \cos x + \cos^2 x$$

$$2\sin^2 x + 2\cos^2 x$$

$$2(\sin^2 x + \cos^2 x)$$

$$2(1)$$

$$\boxed{2}$$

$$\textcircled{9} \frac{1 + \boxed{\tan x}}{1 + \boxed{\cot x}} = \boxed{\tan x}$$

$$\frac{1 + \frac{\sin x}{\cos x}}{1 + \frac{\cos x}{\sin x}}$$

$$\frac{1 + \frac{\cos x}{\sin x}}{1 + \frac{\sin x}{\cos x}}$$

$$\frac{\cos x + \sin x}{\cos x} \div \frac{\sin x + \cos x}{\sin x}$$

$$\frac{(\cancel{\sin x + \cos x})}{\cos x} \times \frac{\sin x}{(\cancel{\sin x + \cos x})}$$

$$\boxed{\frac{\sin x}{\cos x}}$$

$$\boxed{\frac{\sin x}{\cos x}}$$

$$\textcircled{12} \boxed{\csc x} (\sin x + \cos x) = 1 + \cot x$$

$$\left(\frac{1}{\sin x}\right) (\sin x + \cos x)$$

$$\boxed{\frac{\sin x + \cos x}{\sin x}}$$

$$\frac{1}{1} + \frac{\cos x}{\sin x}$$

$$\boxed{\frac{\sin x + \cos x}{\sin x}}$$

$$\textcircled{15} \tan^3 x \sec^2 x - \tan^3 x = \boxed{\tan^5 x}$$

$$\tan^3 x (\sec^2 x - 1)$$

$$(\tan^3 x)(\tan^2 x)$$

$$\boxed{\tan^5 x}$$

$$\textcircled{5} \quad \tan^3 x \sec^2 x - \tan^3 x = \tan^5 x$$

$$\left(\frac{\sin^3 x}{\cos^3 x}\right) \left(\frac{1}{\cos^2 x}\right) - \frac{\sin^3 x}{\cos^3 x}$$

$$\frac{\sin^3 x}{\cos^5 x} - \frac{\sin^3 x}{\cos^3 x}$$

$$\frac{\sin^3 x - \sin^3 x \cos^2 x}{\cos^5 x}$$

$$\frac{\sin^3 x (1 - \cos^2 x)}{\cos^5 x}$$

$$\frac{(\sin^3 x)(\sin^2 x)}{\cos^5 x}$$

$$\frac{\sin^5 x}{\cos^5 x}$$

$$\frac{\sin^5 x}{\cos^5 x}$$

$$\textcircled{3} \quad \cos A + \tan A \sin A = \sec A$$

$$\cos A + \left(\frac{\sin A}{\cos A}\right) \sin A$$

$$\frac{\cos A}{1} + \frac{\sin^2 A}{\cos A}$$

$$\frac{\cos^2 A + \sin^2 A}{\cos A}$$

$$\frac{1}{\cos A}$$

$$\frac{1}{\cos A}$$

Trig Identities #4

⑮ $\frac{\sin^4 \theta - \cos^4 \theta}{\sin^2 \theta \cos^2 \theta - \cos^4 \theta} = \frac{\sec^2 \theta}{\cot^2 \theta}$

Diff of Squares (red arrow pointing to the numerator)

Pythagorean (green arrow pointing to the numerator)

Common factor (red arrow pointing to the denominator)

$$\frac{(\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta - \cos^2 \theta)}{\cos^2 \theta (\sin^2 \theta - \cos^2 \theta)}$$

$$\frac{1}{\cos^2 \theta}$$

$$\frac{1}{\sin^2 \theta} \div \frac{\cos^2 \theta}{\sin^2 \theta}$$

$$\frac{1}{\sin^2 \theta} \times \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$\frac{1}{\cos^2 \theta}$$

⑯ $\sec^2 \theta - \sin^2 \theta = \cos^2 \theta + \tan^2 \theta$

$\tan^2 \theta + 1 - (1 - \cos^2 \theta)$

$\tan^2 \theta + 1 - 1 + \cos^2 \theta$

$$\cos^2 \theta + \tan^2 \theta$$

⑰ $\sec^2 \theta - \sin^2 \theta = \cos^2 \theta + \tan^2 \theta$

$$\sec^2 \theta - \tan^2 \theta = \sin^2 \theta + \cos^2 \theta$$

$$1 = 1$$

Bonus

Prove the following identity:

$$\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = 2 \sec x$$

Foil \rightarrow

$$\frac{(1 + \sin x)(1 + \sin x) + \cos^2 x}{\cos x(1 + \sin x)}$$

Pythagorean = 1

$$\frac{1 + 2\sin x + \sin^2 x + \cos^2 x}{\cos x(1 + \sin x)}$$

Factor \rightarrow

$$\frac{2 + 2\sin x}{\cos x(1 + \sin x)}$$

$$\frac{2(1 + \sin x)}{\cos x(1 + \sin x)}$$

$$\frac{2}{\cos x}$$

$$2 \left(\frac{1}{\cos x} \right)$$

$$\frac{2}{\cos x}$$

Quiz & Homework