Questions from homework

① i) $t_1 = 99 \qquad t_n = \frac{1}{3}(t_{n-1})$

$t_2 = \frac{1}{3}(t_1) = \frac{1}{3}(99) = 33$

$t_3 = \frac{1}{3}(t_2) = \frac{1}{3}(33) = 11$

$t_4 = \frac{1}{3}(t_3) = \frac{1}{3}(11) = \frac{11}{3}$

$t_5 = \frac{1}{3}(t_4) = \frac{1}{3}\left(\frac{11}{3}\right) = \frac{11}{9}$

④ 5, 4, 1, 3, -2, 5

$t_1 = 5$

$t_2 = 4$

$t_n = t_{n-2} - t_{n-1}$

$t_3 = t_1 - t_2$

$t_3 = 5 - 4$

$t_3 = 1$

⑨ $8x^5, 4x^4, 2x^3, x^2, \ldots$

$t_1 = 8x^5 \qquad t_n = \frac{1}{2x}(t_{n-1})$

$t_5 = \frac{1}{2x}(t_4)$

$= \frac{1}{2x}(x^3)$

$= \frac{x^3}{2x}$

$= \frac{x}{2}$ or $\frac{1}{2}x$

Sequences

Find the first 5 terms of the following sequences:

$t_n = 3^n$

$t_1 = 3^1 = 3$

$t_2 = 3^2 = 9$

$t_3 = 3^3 = 27$

$t_4 = 3^4 = 81$

$t_5 = 3^5 = 243$

$t_n = n + 5$

$t_1 = 1 + 5 = 6$

$t_2 = 2 + 5 = 7$

$t_3 = 3 + 5 = 8$

$t_4 = 4 + 5 = 9$

$t_5 = 5 + 5 = 10$

$t_n = (n+2)(n-1)$

$t_1 = (3)(0) = 0$

$t_2 = (4)(1) = 4$

$t_3 = (5)(2) = 10$

$t_4 = (6)(3) = 18$

$t_5 = (7)(4) = 28$

Arithmetic Sequences

Ex: 2, 5, 8, 11, 14

- The difference between each term is constant.
- In the sequence 2, 5, 8, 11, 14. the difference between each term is 3.
- The difference is called **"d".** $d = t_2 - t_1$
- The first term is called **"a"** or **"t_1".**
- The second term is called **"t_2".**
- The last term or an indicated term is called **"t_n".** (general term)
- The position of a term or the number of terms is called **"n".**

Arithmetic Sequences

To find any given term in an arithmetic sequence we use the following formula:

last term ↙ $t_n = a + (n-1)d$ ← common difference

# of terms ↙ n

a ↑ first term

Example I.

Find the indicated term of the following sequence

2, 4, 6...

$a = 2$

$d = t_2 - t_1$
$= 4 - 2$
$= 2$

$n = 7$

$t_7 = 2 + (7-1)2$
$= 2 + (6)(2)$
$= 2 + 12$
$= 14$

$n = 50$

$t_{50} = 2 + (50-1)2$
$= 2 + (49)(2)$
$= 2 + 98$
$= 100$

We can also determine the number of terms in the sequence.

$$t_n = a + (n-1)d$$

Example II.

How many terms are in the following sequences? (Solve for "n")

1, 3, 5,... 71

$a = 1$

$d = 2$

$t_n = 71$ (last term)

$n = ?$

$71 = 1 + (n-1)2$

$71 = 1 + 2n - 2$

$71 = 2n - 1$

$72 = 2n$

$36 = n$

$x,\ x+3,\ x+6,\ \ldots\ ,x+33$

$a = x$

$d = 3$

$t_n = x+33$

$n = ?$

$x+33 = x + (n-1)3$

$x+33 = x + 3n - 3$

$33 = 3n - 3$

$36 = 3n$

$12 = n$

Find "a", "d", and "t_n" for the following sequence

$$t_5 = 16,\ t_8 = 25$$

$t_5 = a+(5-1)d$ $\quad$ $t_8 = a+(8-1)d$

$t_5 = a+4d$ $\quad$ $t_8 = a+7d$

$a+4d=16$ $\quad$ $a+7d=25$ $\quad$ 2x2 system

$$\begin{array}{r} a+4d=16 \\ \leftrightarrow\ a+7d=25 \\ \hline -3d=-9 \\ d=3 \end{array}$$

$a+4d=16$

$a+4(3)=16$

$a+12=16$

$a=4$

$t_n = a+(n-1)d$

$t_n = 4+(n-1)3$

$t_n = 4+3n-3$

$t_n = 3n+1$

Homework

#1
#2
#3
#4
#6
#7
#9