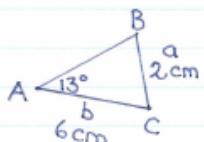


## SOLUTIONS $\Rightarrow$ 4.3 The Ambiguous Case (WORKBOOK)

- I. Determine whether each description of a triangle involves the SSA situation.

a) In  $\triangle ABC$ ,  $\angle A = 13^\circ$ ,  $a = 2\text{cm}$ , and  $b = 6\text{cm}$ .

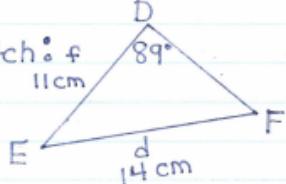
Sketch:



This is a SSA situation.

b) In  $\triangle DEF$ ,  $\angle D = 89^\circ$ ,  $d = 14\text{cm}$ , and  $f = 11\text{cm}$

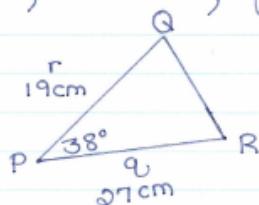
Sketch:



This is a SSA situation.

c) In  $\triangle PQR$ ,  $\angle P = 38^\circ$ ,  $q = 27\text{cm}$ , and  $r = 19\text{cm}$ .

Sketch:



This is not a SSA situation.  
(SAS)

3. Calculate the height of each triangle, to the nearest tenth of a centimeter. Determine the number of triangles that are possible. Give your reasons.

a) In  $\Delta RST$ ,  $\angle R = 103^\circ$ ,  $r = 16\text{ cm}$ , and  $s = 9\text{ cm}$ .

① This is a SSA situation. (Matching Pair).

② We can skip this step

Since  $\angle R$  is obtuse.

③ Since  $\angle R$  is obtuse and  $r > s$ , one obtuse triangle is possible.

b) In  $\Delta XYZ$ ,  $\angle X = 50^\circ$ ,  $x = 5.2 \text{ cm}$ , and  $z = 7.1 \text{ cm}$ .

① This is a SSA situation. (Matching Pair).

$$\begin{aligned} ② h &= b \sin A \\ \hookrightarrow h &= z \sin X \\ h &= 7.1 \sin 50^\circ \\ h &= 5.4 \text{ cm} \end{aligned}$$

③ Since  $\angle X$  is acute  
and  $x < h$ , no triangle  
is possible.

c) In  $\Delta ABC$ ,  $\angle A = 74^\circ$ ,  $a = 28.0 \text{ cm}$ , and  $b = 28.9 \text{ cm}$

① This is a SSA situation. (Matching Pair).

$$\begin{aligned} ② h &= b \sin A \\ h &= 28.9 \sin 74^\circ \\ h &= 27.8 \text{ cm} \end{aligned}$$

③ Since  $\angle A$  is acute  
and  $h < a < b$ , two  
triangles are possible.

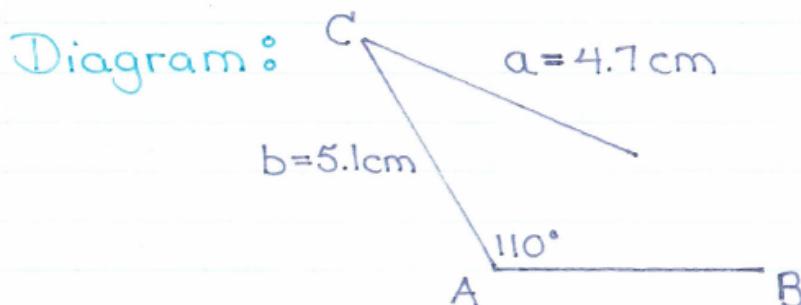
4. Given each set of measurements for  $\triangle ABC$ , determine the number of triangles that are possible. Draw a diagram to support your answer.

a)  $\angle A = 110^\circ$ ,  $a = 4.7\text{cm}$ , and  $b = 5.1\text{cm}$ .

① This is a SSA situation. (Matching Pair).

② We can skip this step since  $\angle A$  is obtuse.

③ Since  $\angle A$  is obtuse and  $a < b$ , no triangle is possible.



b)  $\angle A = 50^\circ$ ,  $a = 6.3\text{cm}$ , and  $b = 8.2\text{cm}$ .

① This is a SSA situation. (Matching Pair)

②  $h = b \sin A$

$$h = 8.2 \sin 50^\circ$$

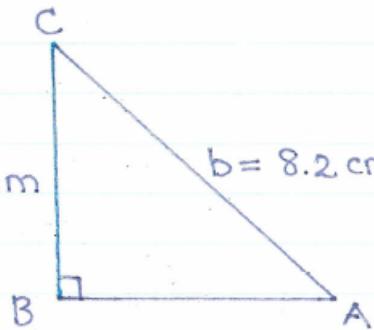
$$h = 6.3\text{ cm}$$

③ Since  $\angle A$  is acute  
and  $a = h$ , one right  
triangle is possible.

Diagram:

$$a = 6.3\text{ cm}$$

$$b = 8.2\text{ cm}$$



5. In  $\Delta XYZ$ ,  $\angle X = 67^\circ$ ,  $x = 3.2\text{m}$ , and  $y = 3.4\text{m}$

a) Determine a possible measure for  $\angle Y$ , to the nearest degree.

$$\frac{\sin Y}{y} = \frac{\sin X}{x}$$

$$\frac{\sin Y}{3.4} = \frac{\sin 67^\circ}{3.2}$$

$$\frac{3.2 \sin Y}{3.2} = \frac{3.4 \sin 67^\circ}{3.2}$$

$$\sin Y = 0.9780$$

$$Y = \sin^{-1}(0.9780)$$

$$Y = 78^\circ$$

b) State another possible value for  $\angle Y$ . Give one reason why this is a possible value.

$$\sin \theta = \sin(180^\circ - \theta)$$

$$\sin \theta = \sin(180^\circ - 78^\circ)$$

$$\sin \theta = \sin 102^\circ$$

Since  $\angle X$  is acute,  $\angle Y$  could also be obtuse.  
 $(102^\circ)$

c) Which of these possible values for  $\angle Y$  corresponds to a possible triangle?  
Explain.

$$h = b \sin A$$

$$\hookrightarrow h = y \sin X$$

$$h = 3.4 \sin 67^\circ$$

$$h = 3.4(0.9205)$$

$$h = 3.1 \text{ m}$$

Since  $\angle X$  is acute and  $h < x < y$ , two triangles are possible. Therefore both values for  $\angle Y$  correspond to possible triangles.

## MULTIPLE CHOICE

6. In  $\triangle MNP$ ,  $n = 4.5 \text{ cm}$  and  $\angle M = 35^\circ$ .  
What is the height of the triangle from base p?

$$\hookrightarrow h = b \sin A$$

$$\hookrightarrow h = n \sin M$$

$$h = 4.5 \sin 35^\circ$$

$$h = 4.5(0.5736)$$

$$h = 2.6 \text{ cm}$$

Option  $\Rightarrow$  "C"

7. In  $\Delta PQR$ ,  $\angle P = 108^\circ$ ,  $q = 4.9\text{m}$ , and  $p = 4.5\text{m}$ .

Which statement is true for this set of measurements?

Since  $\angle P$  is obtuse and  $p < q$ , no triangle is possible.

Option  $\Rightarrow$  "B" This is an SSA situation;  
no triangle is possible.

8. Which set of measurements can produce two possible triangles?

\* We need to check option "A" and option "B".  
The solution cannot be option "C" since  $\angle A$  is obtuse.

"A"

$$h = b \sin A$$

$$h = 4.0 \sin 62^\circ$$

$$h = 4.0(0.8829)$$

$$h = 3.5 \text{ m}$$

Since  $\angle A$  is acute and  $a < h$ , no triangle is possible.

"B"

$$h = b \sin A$$

$$h = 7.4 \sin 52^\circ$$

$$h = 7.4(0.7880)$$

$$h = 5.8 \text{ m}$$

Since  $\angle A$  is acute and  $h < a < b$ , two triangles are possible.

Option  $\Rightarrow$  "B"

9. In obtuse  $\triangle ABC$ ,  $\angle B = 24^\circ$ ,  $b = 18\text{cm}$ , and  $a = 22\text{ cm}$ . Calculate the measure of  $\angle A$ , to the nearest degree. Is there more than one possible answer? Explain how you know.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{22} = \frac{\sin 24^\circ}{18}$$

$$\frac{18 \sin A}{18} = \frac{22 \sin 24^\circ}{18}$$

$$\sin A = 0.4971$$

$$A = \sin^{-1}(0.4971)$$

$$A = 30^\circ$$

$$\sin 30^\circ = \sin(180^\circ - \theta)$$

$$\sin 30^\circ = \sin(180^\circ - 30^\circ)$$

$$\sin 30^\circ = \sin 150^\circ$$

$\angle A$  could be  $30^\circ$  or  $150^\circ$  since there are 2 possible triangles.

$$h = b \sin A$$

$$\hookrightarrow h = a \sin B$$

$$h = 22 \sin 24^\circ$$

$$h = 22 (0.4067)$$

$$h = 8.9\text{ cm}$$

( $h < b < a$ ; 2 possible triangles)

10. In  $\triangle DEF$ ,  $\angle D = 97^\circ$ ,  $d = 5.3\text{cm}$ , and  $e = 4.8\text{cm}$ . Calculate the measure of  $\angle E$ , to the nearest degree. Is there more than one possible answer? Explain how you know.

$$\frac{\sin E}{e} = \frac{\sin D}{d}$$

$$\frac{\sin E}{4.8} = \frac{\sin 97^\circ}{5.3}$$

$$\frac{5.3 \sin E}{5.3} = \frac{4.8 \sin 97^\circ}{5.3}$$

$$\sin E = 0.8989$$

$$E = \sin^{-1}(0.8989)$$

$$E = 64^\circ$$

Since  $\angle D$  is obtuse, and  $d > e$ , one obtuse triangle is possible.