

# Warm Up

Which of the following is the inverse of the matrix  $\begin{pmatrix} 4 & -3 \\ 2 & -2 \end{pmatrix}$ ?

[A]  $\begin{pmatrix} -1 & \frac{3}{2} \\ -1 & 2 \end{pmatrix}$

[B]  $\begin{pmatrix} 2 & -1 \\ \frac{3}{2} & -1 \end{pmatrix}$

[C]  $\begin{pmatrix} 1 & -\frac{3}{2} \\ 1 & -2 \end{pmatrix}$

[D]  $\begin{pmatrix} \frac{1}{7} & -\frac{3}{14} \\ \frac{1}{7} & -\frac{2}{7} \end{pmatrix}$

Which system of equations would you use to represent the cost of these two newspaper classified rates?

- The "Daily Gleaner" has a flat rate of \$18 plus 20¢ per word
- The "Times-Transcript" has a flat rate of \$25 plus 10¢ per word

(A)  $C - 18 = 20w$   
 $C - 25 = 10w$

(B)  $18 + C = 20w$   
 $25 + C = 10w$

(C)  $C = 0.2w + 18$   
 $C = 0.1w + 25$

(D)  $C - 18w = 0.2$   
 $C - 25w = 0.1$

# Questions from Homework

$$\textcircled{4} \text{ a) } \begin{matrix} \textcircled{R1} + 4\textcircled{R2} \\ 2\textcircled{R1} + 9\textcircled{R2} \end{matrix} \left[ \begin{array}{cc|cc} 9 & -4 & 1 & 0 \\ -2 & 1 & 0 & 1 \end{array} \right]$$

$$\left[ \begin{array}{cc|cc} 1 & 0 & 1 & 4 \\ 0 & 1 & 2 & 9 \end{array} \right] \leftarrow \text{Inverse}$$

$$\textcircled{5} \text{ c) } \begin{aligned} 3x - 2y + 5z &= 1 \\ 4x + 5y - 3z &= 17 \\ 7x - 3y + 2z &= 36 \end{aligned}$$

$$\left[ \begin{array}{ccc|c} 3 & -2 & 5 & 1 \\ 4 & 5 & -3 & 17 \\ 7 & -3 & 2 & 36 \end{array} \right] \begin{matrix} 3\textcircled{R2} - 4\textcircled{R1} \\ 3\textcircled{R3} - 7\textcircled{R1} \end{matrix} \left[ \begin{array}{ccc|c} 3 & -2 & 5 & 1 \\ 0 & 23 & -21 & 47 \\ 0 & 5 & -29 & 101 \end{array} \right] \begin{matrix} 23\textcircled{R3} - 5\textcircled{R2} \end{matrix} \left[ \begin{array}{ccc|c} 3 & -2 & 5 & 1 \\ 0 & 23 & -21 & 47 \\ 0 & 0 & -522 & 2088 \end{array} \right]$$

$$\begin{aligned} -522z &= 2088 \\ z &= -4 \end{aligned}$$

$$\begin{aligned} 23y - 21z &= 47 \\ 23y - 21(-4) &= 47 \\ 23y + 116 &= 47 \\ 23y &= -69 \\ y &= -3 \end{aligned}$$

$$\begin{aligned} 3x - 2y + 5z &= 1 \\ 3x - 2(-3) + 5(-4) &= 1 \\ 3x + 6 - 20 &= 1 \\ 3x - 14 &= 1 \\ 3x &= 15 \\ x &= 5 \end{aligned}$$

$$\underline{\underline{(5, -3, -4)}}$$

# 11

Similar

**BONUS!!**

Determine  $a$ ,  $b$ , and  $c$  so that the points  $(-1, 5)$ ,  $(2, -1)$ , and  $(3, 13)$  are on the graph of  $f(x) = ax^2 + bx + c$ .

$$y = ax^2 + bx + c$$

$(-1, 5)$

$$5 = a(-1)^2 + b(-1) + c$$

$$5 = a - b + c$$

$$a - b + c = 5$$

$(2, -1)$

$$-1 = a(2)^2 + b(2) + c$$

$$-1 = 4a + 2b + c$$

$$4a + 2b + c = -1$$

$(3, 13)$

$$13 = a(3)^2 + b(3) + c$$

$$13 = 9a + 3b + c$$

$$9a + 3b + c = 13$$

$$a - b + c = 5$$

$$4a + 2b + c = -1$$

$$9a + 3b + c = 13$$

$$\left[ \begin{array}{ccc|c} 1 & -1 & 1 & 5 \\ 4 & 2 & 1 & -1 \\ 9 & 3 & 1 & 13 \end{array} \right] \rightarrow$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & -5 \end{array} \right]$$

$$a = 4$$

$$b = -6$$

$$c = -5$$

$$\textcircled{1} \begin{bmatrix} \underline{2x+y} & \underline{5} \\ \underline{-1} & \underline{y-x} \end{bmatrix} = \begin{bmatrix} \underline{-2} & \underline{5} \\ \underline{-1} & \underline{1} \end{bmatrix}$$

$$\begin{array}{l} 2x+y = -2 \\ y-x = 1 \end{array} \quad (-) \quad \begin{array}{l} 2x+y = -2 \\ -x+y = 1 \\ \hline 3x = -3 \\ \boxed{x = -1} \end{array} \quad \rightarrow \quad \begin{array}{l} 2x+y = -2 \\ 2(-1)+y = -2 \\ -2+y = -2 \\ \boxed{y = 0} \end{array}$$

$$\begin{array}{l}
 \textcircled{4} \text{ a) } 2x+3y+7z=15 \quad 10x+15y+35z=75 \quad 10x+8y-8z=-4 \\
 5x+4y-4z=-2 \quad \text{b) } 10x+8y-8z=-4 \quad \text{c) } -10x+5y+10z=-5 \\
 -2x+y+2z=-1
 \end{array}$$

$$\begin{array}{l}
 7y+43z=79 \quad 13y+2z=-9
 \end{array}$$

$$\begin{array}{l}
 91y+559z=1027 \\
 \text{c) } 91y+14z=-63 \\
 \hline
 545z=1090 \\
 \boxed{z=2}
 \end{array}$$

$$\begin{array}{l}
 13y+2z=-9 \\
 13y+2(2)=-9 \\
 13y+4=-9 \\
 13y=-13 \\
 \boxed{y=-1}
 \end{array}$$

$$\begin{array}{l}
 2x+3y+7z=15 \\
 2x+3(-1)+7(2)=15 \\
 2x-3+14=15 \\
 2x+11=15 \\
 2x=4 \\
 \boxed{x=2}
 \end{array}$$

(2, -1, 2)

$$\textcircled{4} \text{ a) } \begin{array}{l} 2x+3y+7z=15 \\ 5x+4y-4z=-2 \\ -2x+y+2z=-1 \end{array} \quad \left[ \begin{array}{ccc|c} 2 & 3 & 7 & 15 \\ 5 & 4 & -4 & -2 \\ -2 & 1 & 2 & -1 \end{array} \right]$$

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rref([A])
[[1 0 0 2]
 [0 1 0 -1]
 [0 0 1 2]]
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(2, -1, 2)

$$\begin{aligned} \textcircled{5} \text{ a) } & 3x - 4y + 5z = 26 \\ & 6x - 2y - 3z = -39 \\ & x + 3y - 4z = -31 \end{aligned}$$

$$\left[ \begin{array}{ccc|c} 3 & -4 & 5 & 26 \\ 6 & -2 & -3 & -39 \\ 1 & 3 & -4 & -31 \end{array} \right] \begin{array}{l} 2R - R \\ 3R - R \end{array} \left[ \begin{array}{ccc|c} 3 & -4 & 5 & 26 \\ 0 & -6 & 13 & 91 \\ 0 & 13 & -17 & -119 \end{array} \right] \begin{array}{l} 13R + 6R \end{array} \left[ \begin{array}{ccc|c} 3 & -4 & 5 & 26 \\ 0 & -6 & 13 & 91 \\ 0 & 0 & 67 & 469 \end{array} \right]$$

$$67z = 469$$

$$\boxed{z = 7}$$

$$\begin{aligned} -6y + 13z &= 91 \\ -6y + 13(7) &= 91 \\ -6y + 91 &= 91 \\ -6y &= 0 \\ \boxed{y = 0} \end{aligned}$$

$$\begin{aligned} 3x - 4y + 5z &= 26 & (-3, 0, 7) \\ 3x - 4(0) + 5(7) &= 26 \\ 3x + 35 &= 26 \\ 3x &= -9 \\ \boxed{x = -3} \end{aligned}$$

$$\begin{aligned} \textcircled{5} \text{ a) } & 3x - 4y + 5z = 26 \\ & 6x - 2y - 3z = -39 \\ & x + 3y - 4z = -31 \end{aligned}$$

$$\left[ \begin{array}{ccc|c} 3 & -4 & 5 & 26 \\ 6 & -2 & -3 & -39 \\ 1 & 3 & -4 & -31 \end{array} \right]$$

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rref([B]
      [[1 0 0 -3]
       [0 1 0 0]
       [0 0 1 7]])
```

$$\underline{(-3, 0, 7)}$$

# Review Sheet Homework

# 6, 8, 10, 11, 12, 20, 21

⑧ Let  $x$  = investment @ 8%  
Let  $y$  = " @ 10%  
Let  $z$  = " @ 16%

$$x + y + z = 9000$$

$$0.08x + 0.10y + 0.16z = 1160$$

$$-0.08x - 0.10y + 0.16z = 440$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 9000 \\ 0.08 & 0.1 & 0.16 & 1160 \\ -0.08 & -0.1 & 0.16 & 440 \end{array} \right]$$

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rref([A])  
[[1 0 0 2000]  
 [0 1 0 2000]  
 [0 0 1 5000]]
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$$\textcircled{4} \text{ c) } x + y - z = 1$$

$$\boxed{2y + z = 1}$$

$$3x - y - 5z = 1$$

$$3x + 3y - 3z = 3$$

$$\Leftrightarrow \frac{3x - y - 5z = 1}{3x + 3y - 3z = 3}$$

$$\boxed{4y + 2z = 2}$$

$$4y + 2z = 2$$

$$\Leftrightarrow \frac{4y + 2z = 2}{0 = 0}$$

$$\boxed{0 = 0}$$

$$\text{let } \boxed{z = t} \rightarrow$$

$$2y + z = 1$$

$$2y + t = 1$$

$$2y = 1 - t$$

$$\boxed{y = \frac{1-t}{2}}$$

$$x + y - z = 1$$

$$x + \frac{1-t}{2} - t = 1$$

$$x + \frac{1-t}{2} - \frac{2t}{2} = 1$$

$$x + \frac{1-3t}{2} = 1$$

$$x = 1 - \frac{1}{2} + \frac{3t}{2}$$

$$\boxed{x = \frac{1+3t}{2}}$$

$$\left( \frac{1+3t}{2}, \frac{1-t}{2}, t \right)$$

$$\textcircled{4} \text{ b) } \begin{array}{l} x+y-z=1 \\ \boxed{2y+z=3} \\ 3x-y-5z=0 \end{array} \Leftrightarrow \begin{array}{l} 3x+3y-3z=3 \\ \underline{3x-y-5z=0} \\ \boxed{4y+2z=3} \end{array}$$

$$\begin{array}{l} 4y+2z=6 \\ \Leftrightarrow \underline{4y+2z=3} \\ \boxed{0=3} \end{array}$$

No Solution

$$\textcircled{5}a) \quad 3x - 4y + 5z = 26$$

$$6x - 2y - 3z = -39$$

$$x + 3y - 4z = -31$$

$$\left[ \begin{array}{ccc|c} 3 & -4 & 5 & 26 \\ 6 & -2 & -3 & -39 \\ 1 & 3 & -4 & -31 \end{array} \right] \xrightarrow{\substack{R_2 - 2R_1 \\ R_3 - R_1}} \left[ \begin{array}{ccc|c} 3 & -4 & 5 & 26 \\ 0 & 6 & -13 & -91 \\ 0 & 13 & -17 & -119 \end{array} \right] \xrightarrow{6R_3 - 13R_2} \left[ \begin{array}{ccc|c} 3 & -4 & 5 & 26 \\ 0 & 6 & -13 & -91 \\ 0 & 0 & 67 & 469 \end{array} \right]$$

$$67z = 469 \rightarrow \boxed{z = 7}$$

$$6y - 13z = -91$$

$$6y - 13(7) = -91$$

$$6y - 91 = -91$$

$$6y = 0$$

$$\boxed{y = 0}$$

$$3x - 4y + 5z = 26$$

$$3x - 4y + 5z = 26$$

$$3x - 4(0) + 5(7) = 26$$

$$3x - 0 + 35 = 26$$

$$3x = -9$$

$$\boxed{x = -3}$$

$$\begin{aligned} \textcircled{5} \text{ a) } & 3x - 4y + 5z = 26 \\ & 6x - 2y - 3z = -39 \\ & x + 3y - 4z = -31 \end{aligned}$$

$$\left[ \begin{array}{ccc|c} 3 & -4 & 5 & 26 \\ 6 & -2 & -3 & -39 \\ 1 & 3 & -4 & -31 \end{array} \right] \begin{array}{l} R_2 - 2R_1 \\ R_3 - R_1 \end{array} \left[ \begin{array}{ccc|c} 3 & -4 & 5 & 26 \\ 0 & 6 & -13 & -91 \\ 0 & 20 & -21 & -147 \end{array} \right] \begin{array}{l} 3R_3 - 10R_2 \end{array} \left[ \begin{array}{ccc|c} 3 & -4 & 5 & 26 \\ 0 & 6 & -13 & -91 \\ 0 & 0 & 67 & 469 \end{array} \right]$$

$$67z = 469$$

$$z = 7$$

$$6y - 13z = -91$$

$$6y - 13(7) = -91$$

$$6y - 91 = -91$$

$$6y = 0$$

$$y = 0$$

$$3x - 4y + 5z = 26$$

$$3x - 4(0) + 5(7) = 26$$

$$3x + 35 = 26$$

$$3x = -9$$

$$x = -3$$

$$(-3, 0, 7)$$

$$\begin{aligned} \textcircled{5} \text{ a) } & 3x - 4y + 5z = 26 \\ & 6x - 2y - 3z = -39 \\ & x + 3y - 4z = -31 \end{aligned}$$

$$\left[ \begin{array}{ccc|c} 3 & -4 & 5 & 26 \\ 6 & -2 & -3 & -39 \\ 1 & 3 & -4 & -31 \end{array} \right]$$

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rref([A])
[[1 0 0 -3]
 [0 1 0 0]
 [0 0 1 7]]
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$$\begin{aligned} x &= -3 \\ y &= 0 \\ z &= 7 \end{aligned}$$

$$(-3, 0, 7)$$

x

⑧ Let  $x =$  investment @ 8%.  
Let  $y =$  " @ 10%.  
Let  $z =$  " @ 16%.

$$0.16z = 0.08x + 0.10y + 440$$

$$x + y + z = 9000$$

$$0.08x + 0.10y + 0.16z = 1160$$

$$-0.08x - 0.10y + 0.16z = 440$$

5) a)

$$\left[ \begin{array}{ccc|c} 3 & -4 & 5 & 26 \\ \underline{6} & -2 & -3 & -39 \\ \underline{1} & 3 & -4 & -31 \end{array} \right] \begin{array}{l} R_2 - 2R_1 \\ 3R_3 - R_1 \end{array} \left[ \begin{array}{ccc|c} 3 & -4 & 5 & 26 \\ 0 & 6 & -13 & -91 \\ 0 & \underline{13} & -17 & -119 \end{array} \right] \begin{array}{l} 6R_3 - 13R_2 \end{array} \left[ \begin{array}{ccc|c} 3 & -4 & 5 & 26 \\ 0 & 6 & -13 & -91 \\ 0 & 0 & 67 & 469 \end{array} \right]$$

$$67z = 469$$
$$\boxed{z = 7}$$

$$6y - 13z = -91$$
$$6y - 13(7) = -91$$
$$6y - 91 = -91$$
$$6y = 0$$
$$\boxed{y = 0}$$

$$3x - 4y + 5z = 26$$
$$3x - 4(0) + 5(7) = 26$$
$$3x - 0 + 35 = 26$$
$$3x = -9$$
$$\boxed{x = -3}$$

## Using Algebra

$$\textcircled{4} \text{ a) } \begin{aligned} 2x + 3y + 7z &= 15 \\ 5x + 4y - 4z &= -2 \\ -2x + y + 2z &= -1 \end{aligned}$$

$$\textcircled{4} \text{ b) } \begin{aligned} 2x + 3y + 7z &= 15 \\ -2x + y + 2z &= -1 \end{aligned}$$


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$$4y + 9z = 14$$

$$\textcircled{4} \text{ c) } \begin{aligned} 10x + 8y - 8z &= -4 \\ -10x + 5y + 10z &= -5 \end{aligned}$$


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$$13y + 2z = -9$$

$$\textcircled{4} \text{ d) } \begin{aligned} -8y - 18z &= 28 \\ 117y + 18z &= -81 \end{aligned}$$


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$$109y = -109$$

$$y = -1$$

$$\begin{aligned} 4(-1) + 9z &= 14 \\ -4 + 9z &= 14 \\ 9z &= 18 \\ z &= 2 \end{aligned}$$

$$\begin{aligned} 2x + 3(-1) + 7(2) &= 15 \\ 2x - 3 + 14 &= 15 \\ 2x + 11 &= 15 \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

## Using Matrices

$$\textcircled{4} \text{ a) } \left[ \begin{array}{ccc|c} 2 & 3 & 7 & 15 \\ 5 & 4 & -4 & -2 \\ -2 & 1 & 2 & -1 \end{array} \right] \begin{array}{l} 2R_2 + 5R_1 \\ R_3 + R_1 \end{array} \left[ \begin{array}{ccc|c} 2 & 3 & 7 & 15 \\ 0 & 13 & 2 & -9 \\ 0 & 4 & 9 & 14 \end{array} \right] \begin{array}{l} 13R_3 - 4R_2 \end{array} \left[ \begin{array}{ccc|c} 2 & 3 & 7 & 15 \\ 0 & 13 & 2 & -9 \\ 0 & 0 & 19 & 218 \end{array} \right]$$

$$109z = 218$$

$$z = 2$$

$$\begin{aligned} 13y + 2z &= -9 \\ 13y + 2(2) &= -9 \\ 13y + 4 &= -9 \\ 13y &= -13 \\ y &= -1 \end{aligned}$$

$$\begin{aligned} 2x + 3y + 7z &= 15 \\ 2x + 3(-1) + 7(2) &= 15 \\ 2x - 3 + 14 &= 15 \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

## Using Calculator:

$$\textcircled{4} \text{ a) } \left[ \begin{array}{ccc|c} 2 & 3 & 7 & 15 \\ 5 & 4 & -4 & -2 \\ -2 & 1 & 2 & -1 \end{array} \right]$$

```
rref([A])
[[1 0 0 2]
 [0 1 0 -1]
 [0 0 1 2]]
```

$$\begin{aligned} x &= 2 \\ y &= -1 \\ z &= 2 \end{aligned}$$