

# Warm Up



Evaluate the following limits, if they exist:

1.  $\lim_{x \rightarrow 2} \frac{x-2}{x^3-8}$

$$\lim_{x \rightarrow 2} \frac{\cancel{x-2}}{\cancel{x-2}(x^2+2x+4)}$$

$$\lim_{x \rightarrow 2} \frac{1}{(2^2+2(2)+4)} = \boxed{\frac{1}{12}}$$

2.  $\lim_{x \rightarrow 7} \frac{(\sqrt{x+2}-3)(\sqrt{x+2}+3)}{x-7}$

$$\lim_{x \rightarrow 7} \frac{x+2-9}{(x-7)(\sqrt{x+2}+3)}$$

$$\lim_{x \rightarrow 7} \frac{\cancel{x-7}}{\cancel{x-7}(\sqrt{x+2}+3)}$$

$$\lim_{x \rightarrow 7} \frac{1}{\sqrt{7+2}+3} = \boxed{\frac{1}{6}}$$

3.  $\lim_{h \rightarrow 0} \frac{(a+h)^2 - a^2}{h}$

$$\lim_{h \rightarrow 0} \frac{(a+h-a)(a+h+a)}{h}$$

$$\lim_{h \rightarrow 0} \frac{\cancel{h}(2a+h)}{\cancel{h}}$$

$$\lim_{h \rightarrow 0} 2a+0 = \boxed{2a}$$

## Questions from Homework

$$④ g) \lim_{x \rightarrow 9} \frac{(x-9)(\sqrt{x}+3)}{(\sqrt{x}-3)(\sqrt{x}+3)}$$

$$\lim_{x \rightarrow 9} \frac{\cancel{(x-9)}(\sqrt{x}+3)}{(\sqrt{x}-3)\cancel{(\sqrt{x}+3)}} = \boxed{6}$$

$$⑤ a) \lim_{h \rightarrow 0} \frac{(4+h)^3 - 64}{h}$$

$$\lim_{h \rightarrow 0} \frac{((4+h)-4)((4+h)^2 + 4(4+h) + 16)}{h}$$

$$\lim_{h \rightarrow 0} \frac{\cancel{h}[(4+h)^2 + 4(4+h) + 16]}{\cancel{h}} = \boxed{48}$$

$$⑤ c) \lim_{h \rightarrow 0} \frac{\frac{1}{1+h} - \frac{1}{1}}{h} \quad \text{CD} = (1+h)$$

$$\lim_{h \rightarrow 0} \frac{\frac{1}{1+h} - \frac{1+h}{1+h}}{h}$$

$$\lim_{h \rightarrow 0} \frac{1 - (1+h)}{h(1+h)} \quad * \frac{1-1-h}{h(1+h)}$$

$$\lim_{h \rightarrow 0} \frac{\cancel{-h}}{1+h} \times \frac{1}{\cancel{h}} = \frac{-1}{1+h} = \boxed{-1}$$

$$f) \lim_{h \rightarrow 0} \frac{\frac{1}{(2+h)^2} - \frac{1}{4}}{h} \quad \text{CD} = 4(2+h)^2$$

$$\lim_{h \rightarrow 0} \frac{4 - (2+h)^2}{4h(2+h)^2} \quad \leftarrow \text{Difference of Squares}$$

$$\lim_{h \rightarrow 0} \frac{(2 - (2+h))(2 + (2+h))}{4h(2+h)^2}$$

$$\lim_{h \rightarrow 0} \frac{(2-2-h)(2+2+h)}{4h(2+h)^2}$$

$$\lim_{h \rightarrow 0} \frac{\cancel{(2-2-h)}(4+h)}{4\cancel{h}(2+h)^2} = \frac{-4}{16} = \boxed{-\frac{1}{4}}$$

## The common sense definition of a limit...

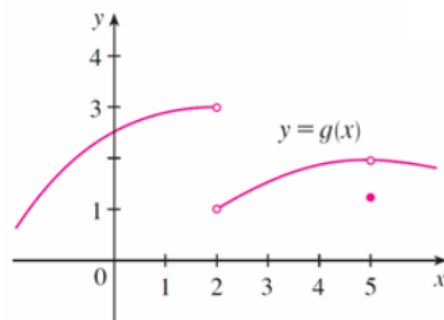


When does a limit exist?



# One-sided limits

Use the graph shown below to evaluate the following limits:



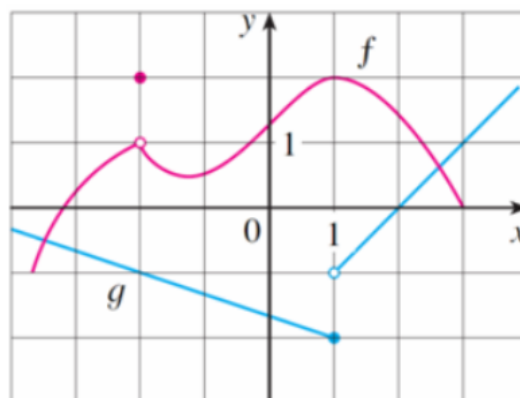
1.  $\lim_{x \rightarrow 2^-} g(x) = \square$       2.  $\lim_{x \rightarrow 2^+} g(x) = \square$       3.  $\lim_{x \rightarrow 2} g(x) = \square$

"as x approaches 2 from the left"      "as x approaches 2 from the right"

4.  $\lim_{x \rightarrow 5^-} g(x) = \square$       5.  $\lim_{x \rightarrow 5^+} g(x) = \square$       6.  $\lim_{x \rightarrow 5} g(x) = \square$

Notice...  $g(5) =$

**Example:**



Evaluate each of the following:

$f(-2) =$        $\lim_{x \rightarrow 1^-} g(x) =$        $g(1) =$

$\lim_{x \rightarrow 1^+} g(x) =$        $\lim_{x \rightarrow 1} g(x) =$        $\lim_{x \rightarrow 1} f(x) =$

$\lim_{x \rightarrow 2} f(x) =$

# Homework

Page 27

# 1, 2, 3

