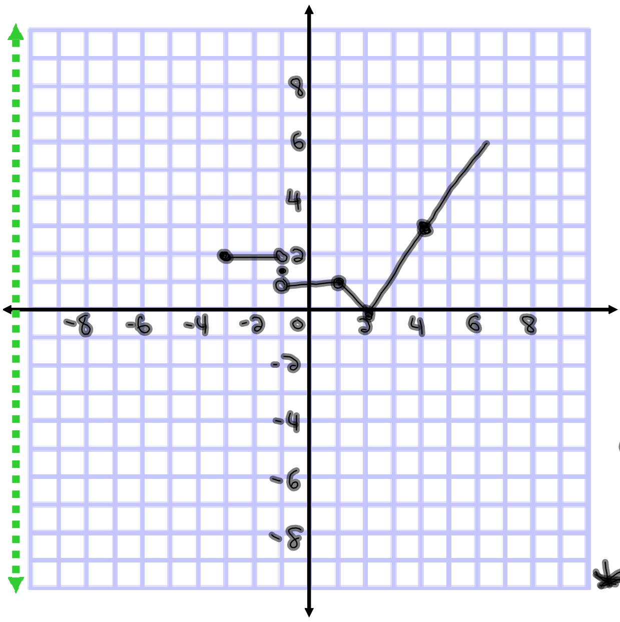
Questions From Homework



a) $\lim_{x\to -3^+} g(x) = 2$

b) $\lim_{x\to -1^-} g(x) = 2$

c) $\lim_{x\to -1^+} g(x) = 1$

d) $\lim_{x\to 4} g(x) = \text{DNE}$

* $g(-1) = 1.5$

③ b) $\lim_{h\to 0} \frac{(2+h)^3-8}{h}$

diff of cubes

$\lim_{h\to 0} \frac{(\overbrace{(2+h)-2}^{2+h-2})((2+h)^2+2(2+h)+4)}{h}$

$\lim_{h\to 0} \frac{\cancel{h}((2+h)^2+2(2+h)+4)}{\cancel{h}} = 4+4+4 = \boxed{12}$

e) $\lim_{x\to 0} \frac{\sqrt{9+x}-3}{x} \cdot \frac{(\sqrt{9+x}+3)}{(\sqrt{9+x}+3)}$

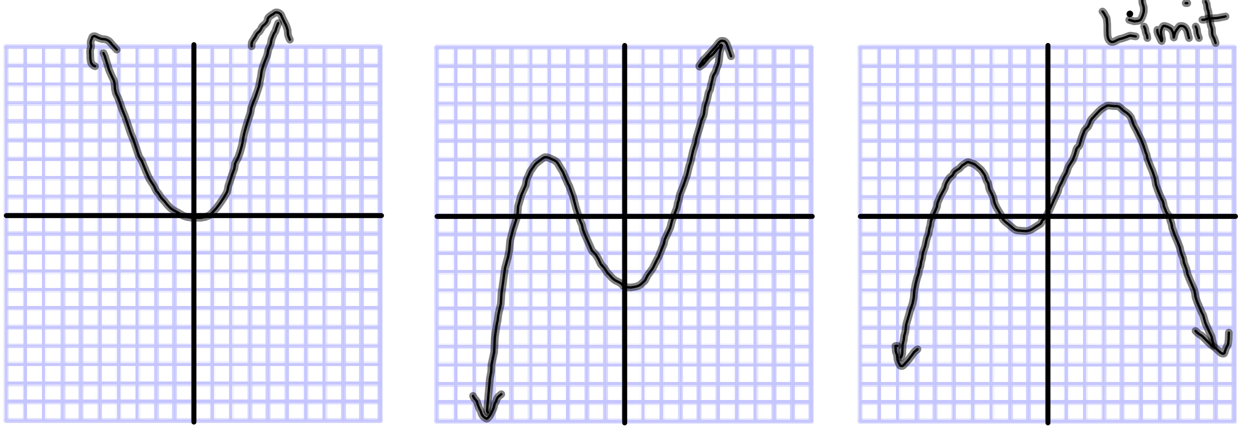
$\lim_{x\to 0} \frac{9+x-9}{x(\sqrt{9+x}+3)}$

$\lim_{x\to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{9+x}+3)} = \boxed{\frac{1}{6}}$

Recall from our previous discussions that ...

1 Theorem $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$

Left Hand Limit = Right Hand Limit



These graphs have limits that exist at every x value and are what we call ***continuous***

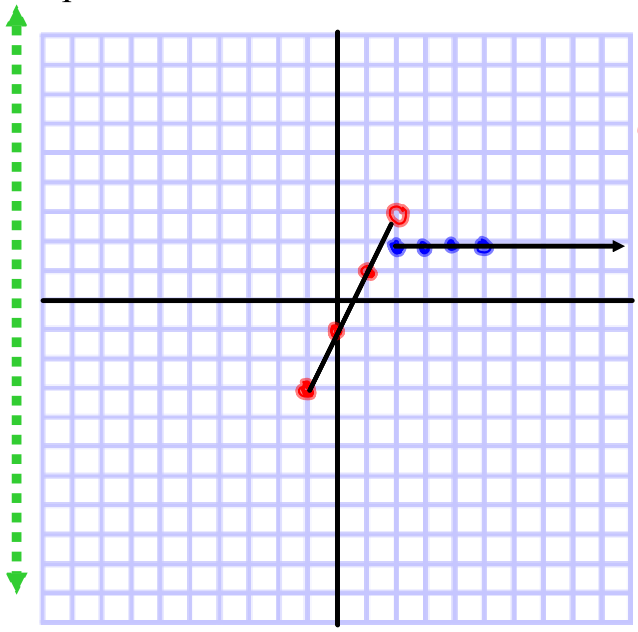
We also want to be able to check limits of piecewise defined functions...

Example:

Domain

$$f(x) = \begin{cases} 2x - 1 & \text{if } -1 \le x < 2 \\ 2 & \text{if } x \ge 2 \end{cases}$$

Graph this function:



$2x-1$

x	$f(x)$
-1	-3
0	-1
1	1
2	3

2

x	$f(x)$
2	2
3	2
4	2
5	2

discontinuous at $x=2$

Evaluate the following limits:

$\lim_{x\to 2^-} f(x) = 3$ $\quad \lim_{x\to 2^+} f(x) = 2$ $\quad \lim_{x\to 2} f(x) = \text{DNE}$

$f(2) = 2$

Continuity

Definition

- We noticed in the preceding section that…
  - the limit of a function as x approaches a

  can often be found simply by…
  - calculating the value of the function at a.
- Functions with this property are called *continuous* at a:

1 Definition A function f is **continuous at a number a** if

$$\lim_{x \to a} f(x) = f(a)$$

- This definition implicitly requires three things if f is continuous at a:
  1. $f(a)$ is defined
     - That is, a is in the domain of f
  2. $f(x)$ has a limit as x approaches a
  3. This limit is actually equal to $f(a)$.

In English! "x-value"
- **Graph must be defined at that point**
- **Limit from left and right must be equal**
- **Limit must be the same as the y value**

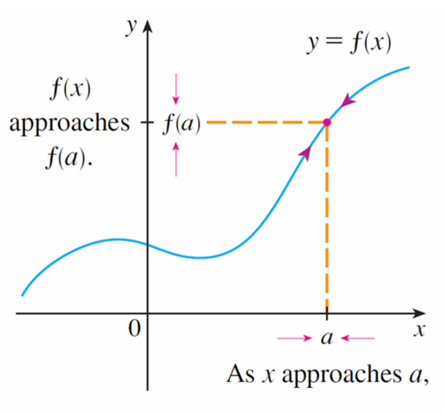


FIGURE 1

Examine the graph shown below
for points of discontinuity...

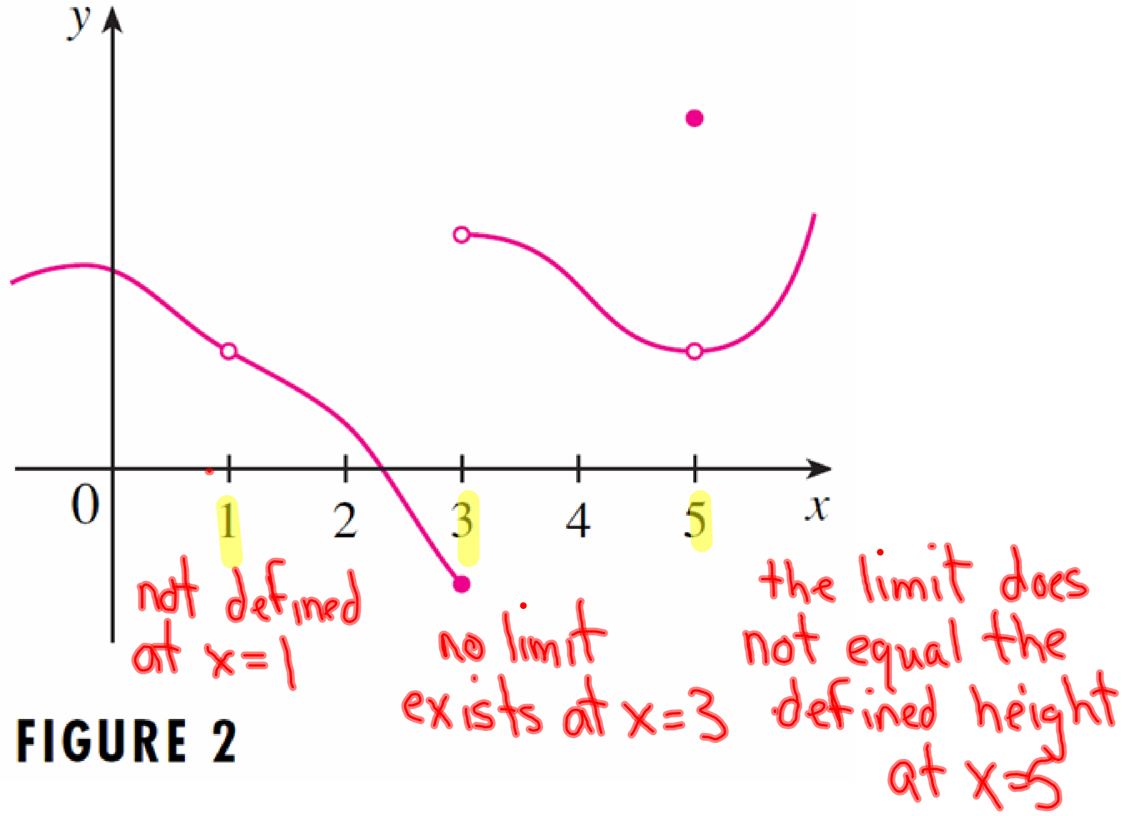


FIGURE 2

- f is discontinuous at 1 because $f(1)$ is not defined…
  - …despite the fact that f has a limit at $a = 1$
- f is also discontinuous at 3, but for a different reason:
  - $f(3)$ is defined, but f has no limit at $a = 3$.
- f has both a value and a limit at 5, but they are different; thus f is discontinuous at 5.

Let's simplify things...

A function whose graph has holes or breaks is considered discontinuous at these particular points. (state the x values)

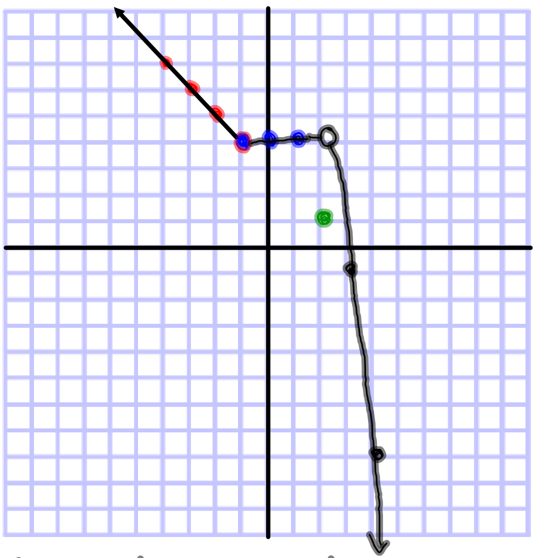
If you have to lift your pencil from the page to sketch the graph, it is discontinuous anywhere you lift your pencil

Examples:

Given the function $f(x) = \begin{cases} 3-x & , \quad \text{if } x<-1 \\ 4 & , \quad \text{if } -1 \le x < 2 \\ 1 & , \quad \text{if } x = 2 \\ 8-x^2 & , \quad \text{if } x > 2 \end{cases}$

(a) Check $f(x)$ for any points of discontinuity. Provide a mathematical reason to validate any point(s) where the function is discontinuous.

(b) Sketch $f(x)$.



$3-x$

x	$f(x)$
o -1	4
-2	5
-3	6

4

x	$f(x)$
• -1	4
0	4
1	4
o 2	4

1

x	$f(x)$
• 2	1

$8-x^2$

x	$f(x)$
o 2	4
3	-1
4	-8

discontinuous at $x=2$

$f(2)=1$

$\lim_{x\to 2} f(x) = 4$

In English!
- **Graph must be defined at that point** ✓
- **Limit from left and right must be equal** ✓
- **Limit must be the same as the y value** ✗

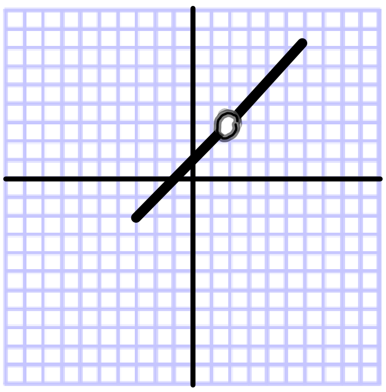
$f(2) \neq \lim_{x\to 2} f(x)$

$\therefore$ The limit does not equal the defined height of the function.

Summary of Continuity:

All of these graphs are discontinuous.

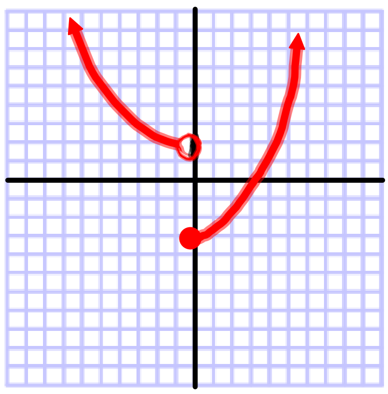
Hole Function



Limit exists but it is not defined

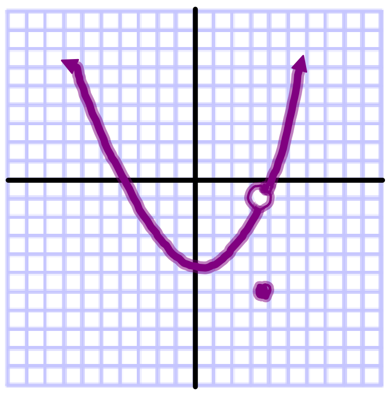
Not Continuous

Step Graph



Limit does not exist

Not Continuous



Limit is not the same as the y value

Not Continuous

Homework

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5, 6, 7, 8 , 9

Try this one...

$$f(x) = \begin{cases} 2 - x^2 & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2x - 1 & \text{if } 1 < x \le 3 \\ (x-4)^2 & \text{if } x > 3 \end{cases}$$

Evaluate:

$\lim_{x \to 1} f(x)$

$\lim_{x \to 3} f(x)$

Given the function $f(x) = \begin{cases} 2 - x^2 & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2x - 1 & \text{if } 1 < x \le 3 \\ (x-4)^2 & \text{if } x > 3 \end{cases}$

(a) Check $f(x)$ for any points of discontinuity. Provide a mathematical reason to validate any point(s) where the function is discontinuous.

(b) Sketch $f(x)$.