Questions from Homework

③ c) $\lim_{x\to 4} \frac{(x-4)}{(\sqrt{x}-2)}\cdot\frac{(\sqrt{x}+2)}{(\sqrt{x}+2)}$

$$\lim_{x\to 4} \frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{(x-4)}} = \boxed{4}$$

d) $\lim_{x\to 0} \frac{\cancel{4(2+x)^2}\cdot\frac{1}{\cancel{(2+x)^2}} - \frac{1}{\cancel{4}}\cdot\cancel{(4)}(2+x)^2}{x(4)(2+x)^2}$ $\quad$ C.D. $= (4)(2+x)^2$

$$\lim_{x\to 0} \frac{4-(2+x)^2}{4x(2+x)^2}$$ ← Diff. of Squares

$$\lim_{x\to 0} \frac{(2-(2+x))(2+(2+x))}{4x(2+x)^2}$$

$$\lim_{x\to 0} \frac{\cancel{(-x)}^{-1}(4+x)}{4\cancel{x}(2+x)^2} = \frac{-4}{16} = \boxed{-\frac{1}{4}}$$

$\hookrightarrow \frac{(-1)(4+0)}{4(1)(2+0)^2}$

② $f(x) = \begin{cases} |x| & x<0 \\ 3 & 0\le x\le 3 \\ (x-3)^2+1 & x>3 \end{cases}$

$|x|$

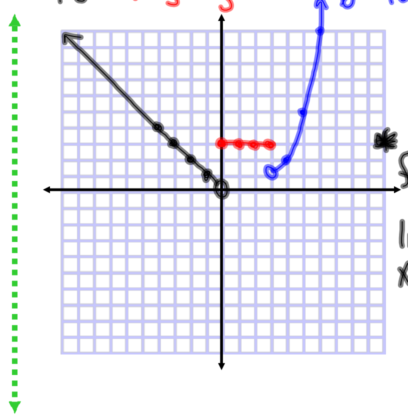
x	$f(x)$
0	0
-1	1
-2	2
-3	3

3

x	$f(x)$
0	3
1	3
2	3
3	3

$(x-3)^2+1$

x	$f(x)$
3	1
4	2
5	5
6	10



Discontinuous at $x = 0, 3$

$f(3) = 3$

$\lim_{x\to 3} f(x) = \text{DNE}$

Limits at Infinity

What exactly is infinity?

- It is the ***process*** of making a value arbitrarily large or small

$+\infty$ ⟶ Positive Infinity...process of becoming arbitrarily large

$-\infty$ ⟶ Negative Infinity...process of becoming arbitrarily small

4 Definition Let f be a function defined on some interval (a, ∞). Then

$$\lim_{x \to \infty} f(x) = L$$

means that the values of $f(x)$ can be made as close to L as we like by taking x sufficiently large.

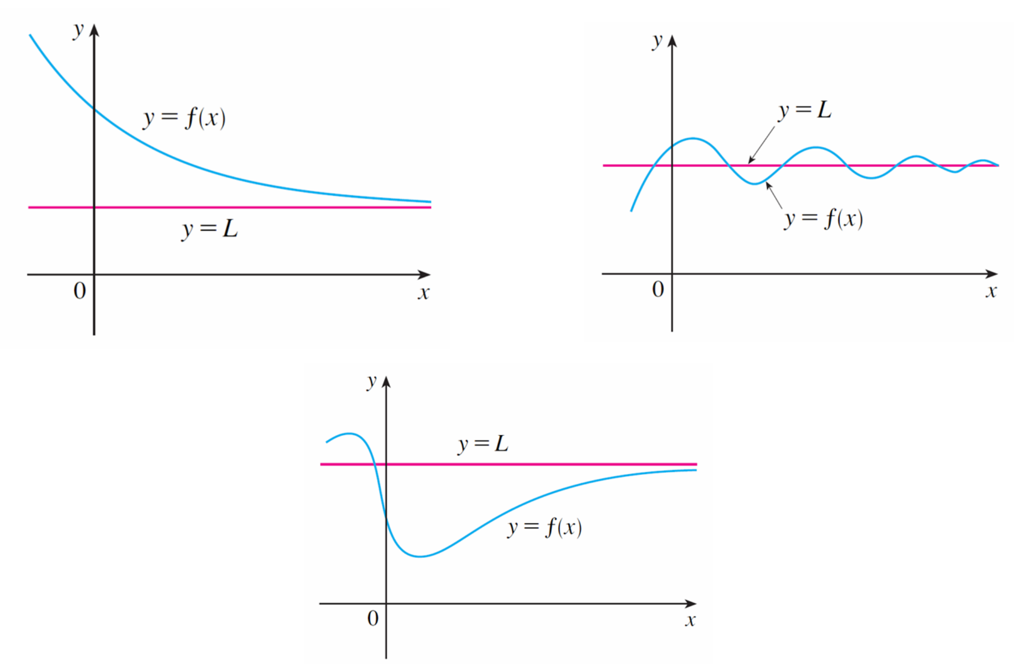
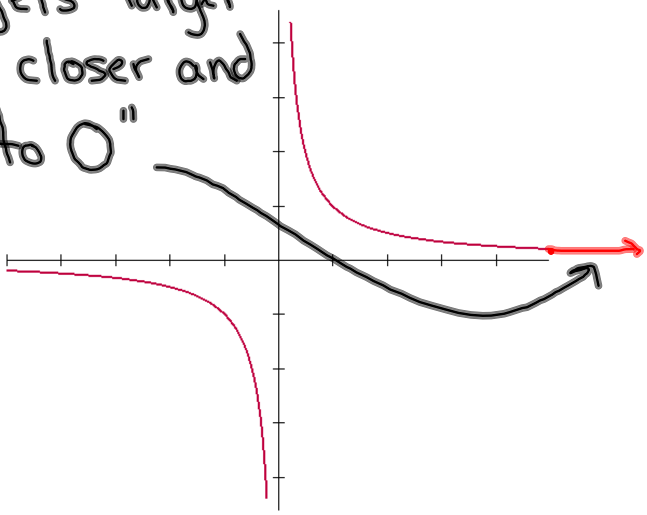


FIGURE 9
Examples illustrating $\lim_{x \to \infty} f(x) = L$

Have a look at these limits...

$$\lim_{x\to\infty} \frac{1}{x} = 0 \qquad \lim_{x\to-\infty} \frac{1}{x} = 0$$

" As x gets larger y gets closer and closer to 0"



In general...

$\boxed{7}$ If n is a positive integer, then

$$\lim_{x\to\infty} \frac{1}{x^n} = 0 \qquad \lim_{x\to-\infty} \frac{1}{x^n} = 0$$

Calculating limits at infinity without using a graph

• Rational Functions

Note: If every term in a rational expression is divided by the same value, the rational expression will still be equal to it's original value

$$\frac{12+8}{6-2} = \frac{20}{4} = \underline{\underline{5}}$$

Divde the numerator and denominator by 2 $\longrightarrow$

$$\frac{6+4}{3-1} = \frac{10}{2} = \underline{\underline{5}}$$

This will be important when evaluating limits for rational functions approaching infinity...

Look at the following example:

$$\lim_{x\to\infty}\frac{3x^2 - x - 2}{5x^2 + 4x + 1} = \lim_{x\to\infty}\frac{\dfrac{3x^2 - x - 2}{x^2}}{\dfrac{5x^2 + 4x + 1}{x^2}} = \lim_{x\to\infty}\frac{3 - \dfrac{1}{x} - \dfrac{2}{x^2}}{5 + \dfrac{4}{x} + \dfrac{1}{x^2}}$$

$$= \frac{3 - 0 - 0}{5 + 0 + 0}$$

$$\boxed{= \frac{3}{5}}$$

$$= 0.6$$

Divide every term by the HIGHEST power that is present in either the numerator or denominator of the rational expression once they are expanded

* For limits at infinity compare the degree of the numerator with the degree of the denominator

This graph below validates our solution:

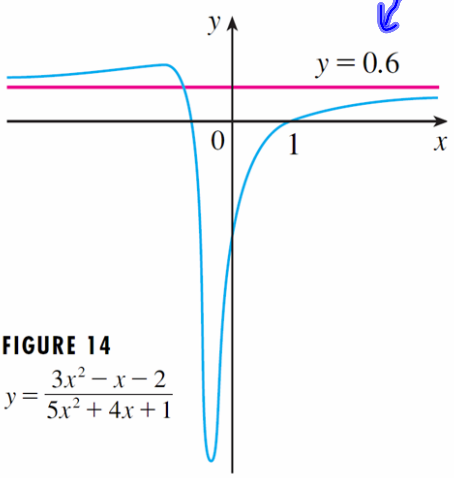


FIGURE 14

$$y = \frac{3x^2 - x - 2}{5x^2 + 4x + 1}$$

- Remember

Limits at Infinity

- **If the highest degree is in the denominator then the *Limit* will be equal to 0**

- **If the highest degree is in the numerator then the *Limit* will not exist.**

- **If the degree is the same in the numerator and denominator then the *Limit* will be equal to the coefficients in front of the highest degree.**

Evaluate the following limit:

$$\lim_{n\to\infty} \frac{n^2 - n}{2n^2 + 1} = \frac{1}{2}$$

$$\lim_{n\to\infty} \frac{1 - n^5}{1 + 2n^5} = \frac{-1}{2}$$

$$\lim_{n\to\infty} \frac{4n}{1} = \text{DNE}$$

$$\lim_{x\to\infty} \frac{-3(x^2 - 4)^2}{3 - 5x^2}$$

$$\lim_{x\to\infty} \frac{-3(x^4 - 8x^2 + 16)}{3 - 5x^2}$$

or $$\lim_{x\to\infty} \frac{-3x^4 + 24x^2 - 48}{3 - 5x^2} = \text{DNE}$$

Homework