Warm Up

Solve the following system of equations:

$4x + 9 = 3y - 6z$
$3z = 10 + 2x + 4y$
$2y = 4z - 11 - 3x$

① $4x - 3y + 6z = -9$
② $2x + 4y - 3z = -10$
③ $3x + 2y - 4z = -11$

$4x - 3y + 6z = -9$
(+) $4x + 8y - 6z = -20$
④ $8x + 5y = -29$

$8x + 16y - 12z = -40$
(−) $9x + 6y - 12z = -33$
⑤ $-x + 10y = -7$

$16x + 10y = -58$
(↔) $-x + 10y = -7$
$17x = -51$
$x = -3$

$8(-3) + 5y = -29$
$-24 + 5y = -29$
$5y = -5$
$y = -1$

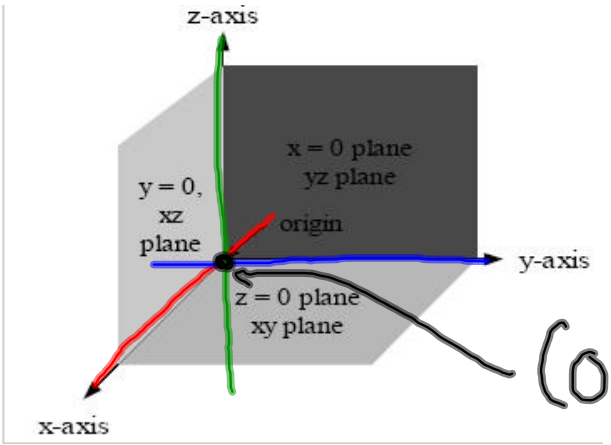
$4(-3) - 3(-1) + 6z = -9$
$-12 + 3 + 6z = -9$
$-9 + 6z = -9$
$6z = 0$
$z = 0$

$(-3, -1, 0)$

Questions from Homework

ALGEBRA OF 3-SPACE

- Coordinate geometry that represents space in **three** dimensions
- Coordinates are in the form of an ordered triplet **(x, y, z)**
- Three planes exist: **xy** plane, **xz** plane, **yz** plane



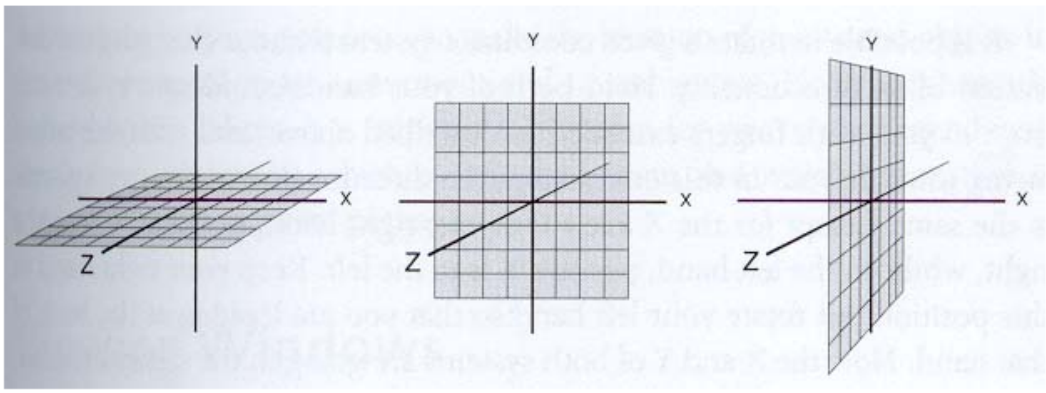
x axis -axis coming "out of the page"

y axis - horzontal axis

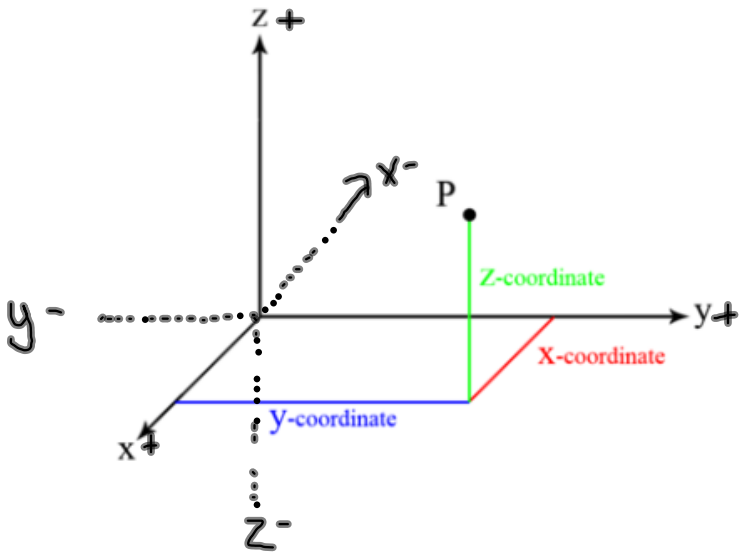
z axis - vertical axis

(0,0,0) → origin

xz plane **xy** plane **yz** plane

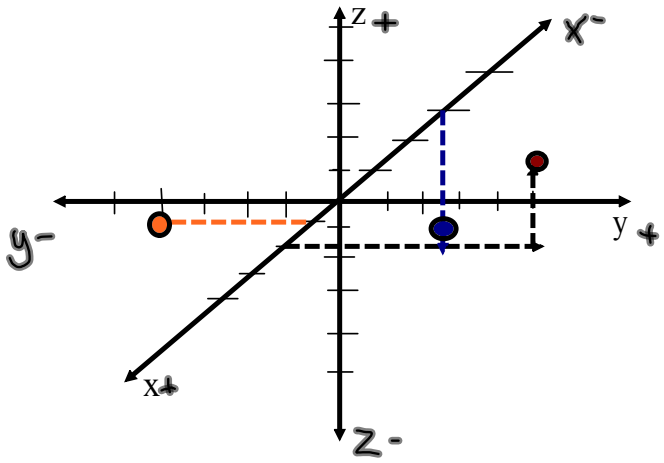


Plotting Points in 3-Space



Plotting points in 3-space...

Ex: a) (2, 6, 3) x, y, z b) (-3, 0, -4) c) (1, -4, 0)

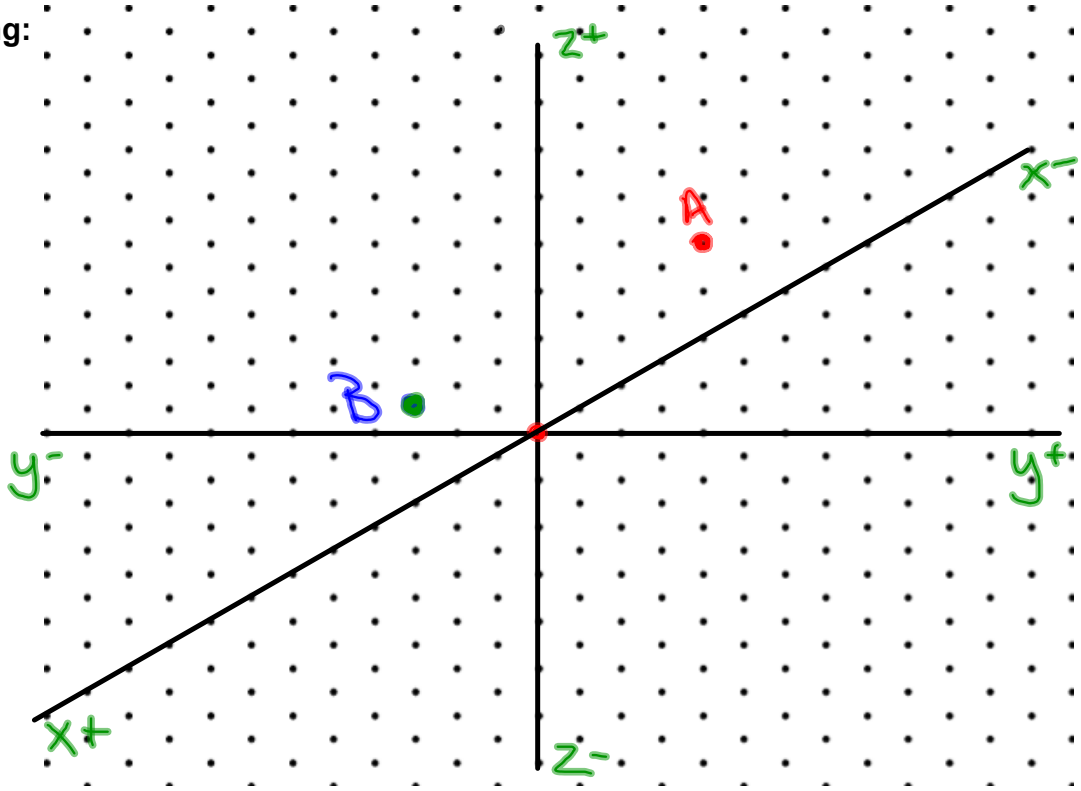


Plot the following:

A (-2, 1, 3)

B (3, 0, 2)

C (-1, -2, 0)



Finding Intercepts in 3D

As in two dimensions...

x intercept can be found when $y = 0$ and $z = 0$

$(x,y,z) \longrightarrow (x,y,z)$

y intercept can be found when $x = 0$ and $z = 0$

$(x,y,z) \longrightarrow (0,y,0)$

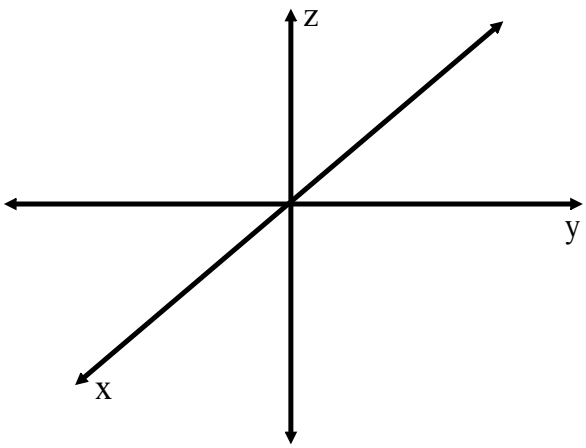
z intercept can be found when $x = 0$ and $y = 0$

$(x,y,z) \longrightarrow (0,0,z)$

Plotting Planes in 3-Space

- Use the **intercept method** to plot the x, y, and z intercepts to form a triangle
- The triangle is part of the plane being sketched

Ex. 2x - y + 3z = 6

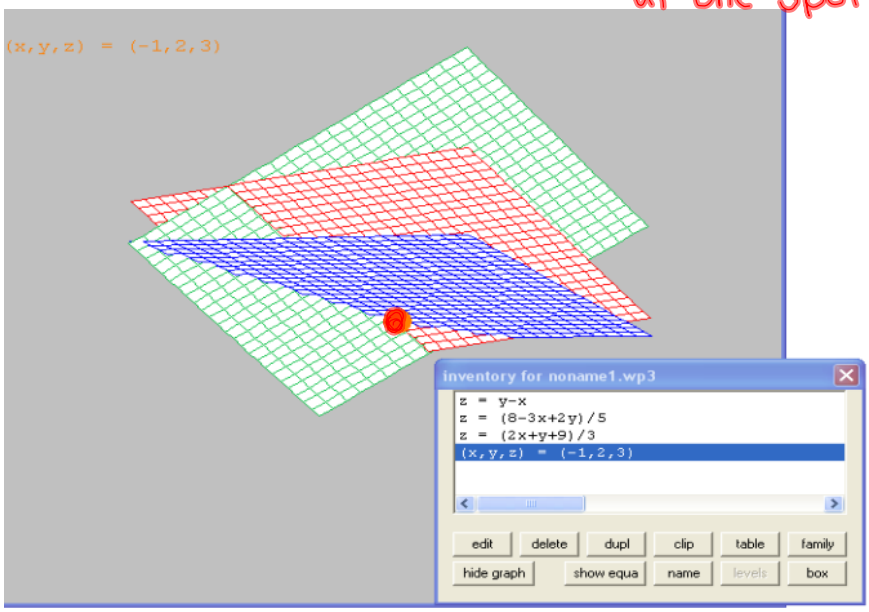


Types of Systems

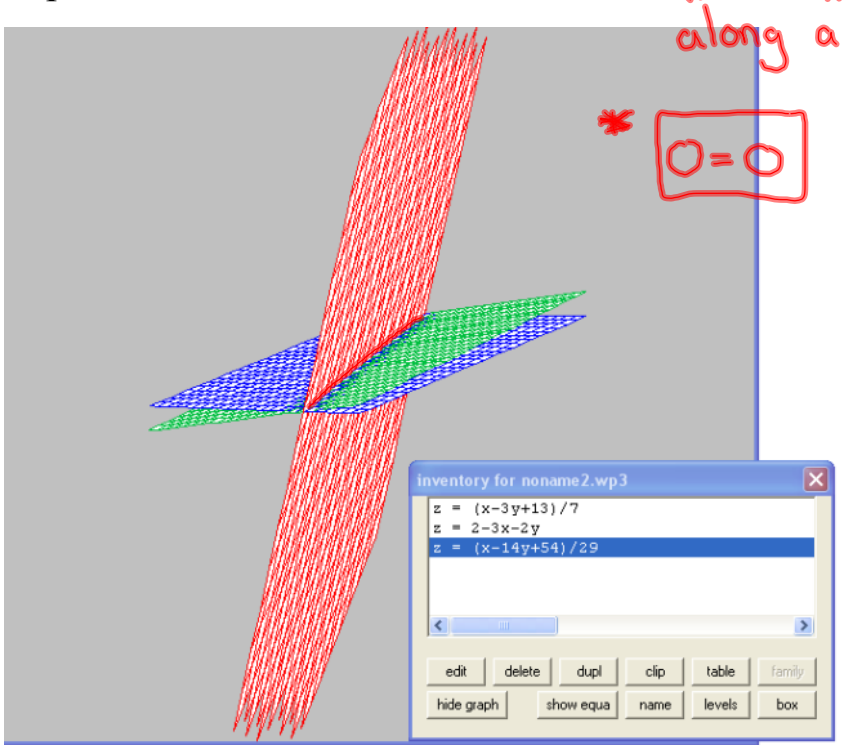
Remember: Looking at **intersecting planes**!

Consistent: At least one solution

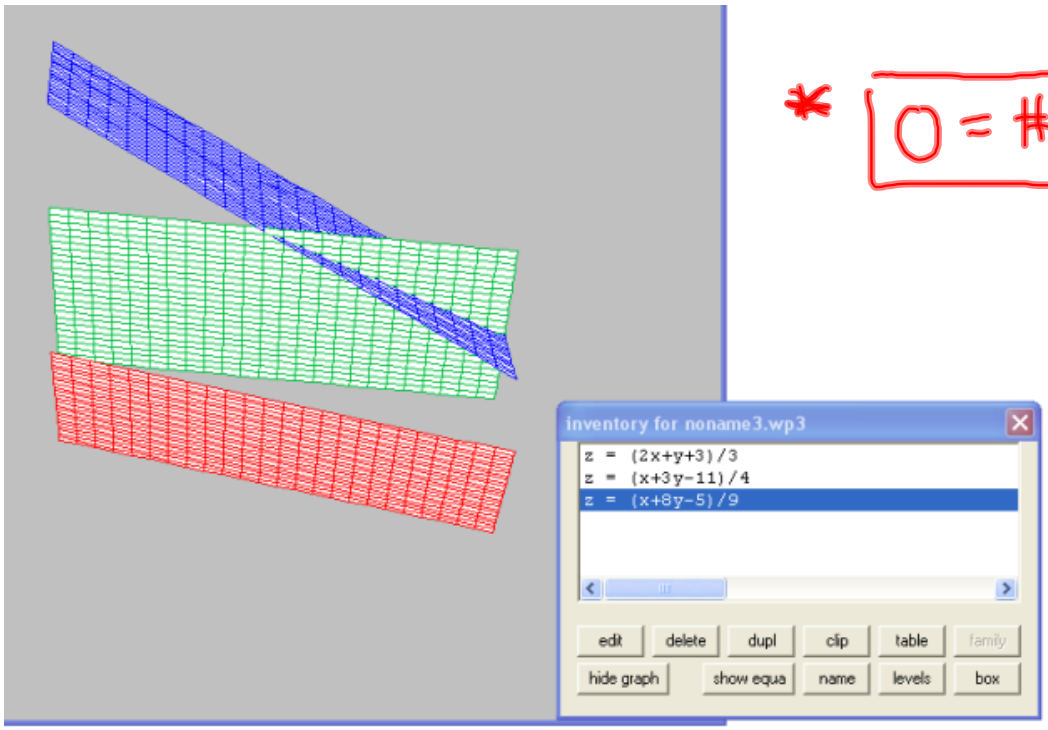
Independent: One Unique solution (Planes intersect at one spot)



Dependent: Infinite number of solutions (Planes intersect along a line)



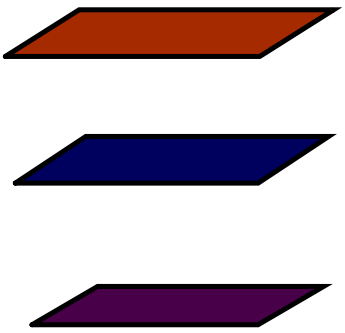
Inconsistent: No Solutions (Planes do not intersect)



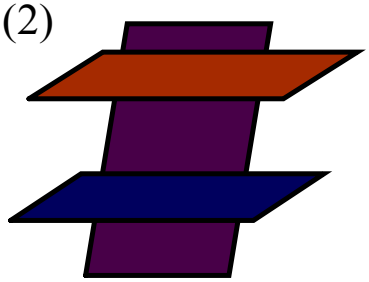
* 0 = #

3 Possible Orientations That Give No Solution...

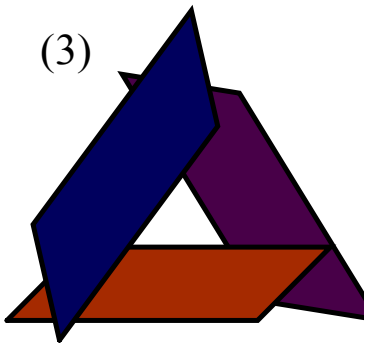
(1)



(2)



(3)



I. Consistent System with an Independent Solution

$x - y + z = 0$
$3x - 2y + 5z = 8$
$2x + y - 3z = -9$

$2x-2y+2z=0$
$\leftrightarrow 3x-2y+5z=8$
$-x-3z=-8$

$3x-2y+5z=8$
$\leftrightarrow 4x+2y-6z=-18$
$7x-z=-10$

$-x-3z=-8$
$\leftrightarrow 2x-3z=-30$
$-2x=22$
$x=-1$

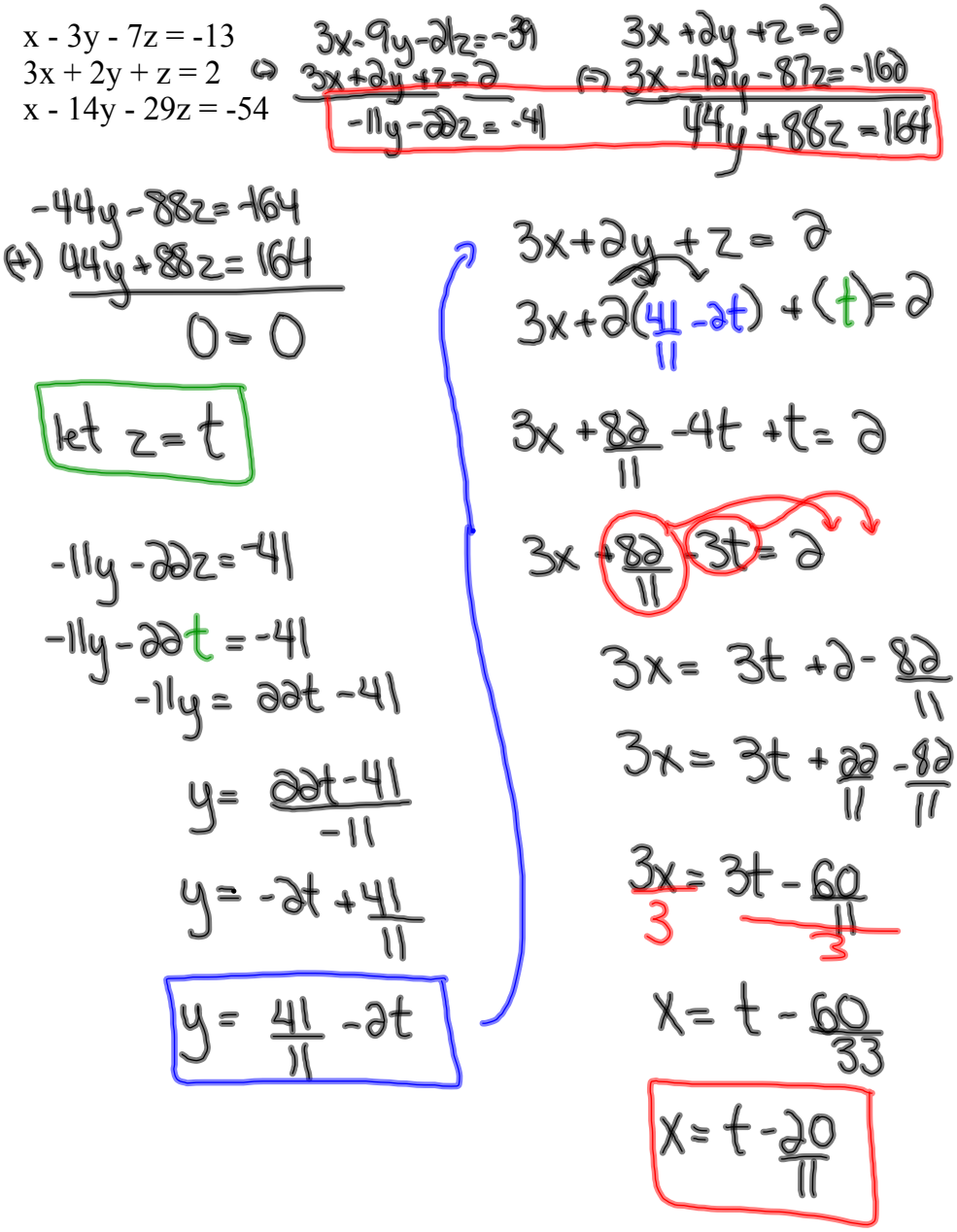
$7x-z=-10$
$7(-1)-z=-10$
$-7-z=-10$
$-z=-3$
$z=3$

$x-y+z=0$
$(-1)-y+(3)=0$
$-y=-2$
$y=2$

$(-1,2,3)$

II. Consistent System with a Dependent Solution
(must create a parametric solution)

$$\begin{aligned} x - 3y - 7z &= -13 \\ 3x + 2y + z &= 2 \\ x - 14y - 29z &= -54 \end{aligned}$$



Write a general solution in terms of a parameter (i.e. z = t). For each value assigned to the parameter there will be one distinct solution.

III. Inconsistent System
(planes do not intersect)

$3x + 2y + z = 3$
$x - 3y + z = 4$
$-6x - 4y - 2z = 1$

$3x+2y+z=3$
$\leftrightarrow \quad x-3y+z=4$
$2x+5y=-1$

$2x-6y+2z=8$
$+ \quad -6x-4y-2z=1$
$-4x-10y=9$

$4x+10y=-2$
$-4x-10y=9$
$0 \neq 7$

No Solution

Homework

Handout: Solving Systems of Equations in 3-Space

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