

29) b) $x^3 - 3x^2 - 33x + 35$ $(x-1)$ is a factor

$$(1)^3 - 3(1)^2 - 33(1) + 35$$

$$1 - 3 - 33 + 35$$

$$0$$

31) c) $x^3 + 4x^2 - x - 10$ Factor Theorem

$$\begin{array}{r} \underline{x-3} \\ \hline x^4 + x^3 - 13x^2 - 7x + 30 \\ - (x^4 - 3x^3) \\ \hline 4x^3 - 13x^2 \\ - (4x^3 - 12x^2) \\ \hline -x^2 - 7x + 30 \\ - (-x^2 + 3x) \\ \hline -10x + 30 \\ - (-10x + 30) \\ \hline 0 \end{array}$$

$$\begin{array}{r} \underline{x+2} \\ \hline x^3 + 4x^2 - x - 10 \\ - (x^3 + 2x^2) \\ \hline 2x^2 - x - 10 \\ - (2x^2 + 4x) \\ \hline -5x - 10 \\ + (5x + 10) \\ \hline 0 \end{array}$$

Factors:

$$(x-3)(x+2)(x^2+2x-5)$$

32 a)

(i)
x-value

2

$$\begin{array}{r} 1 \quad -4 \quad 1 \quad 6 \\ \quad \quad 2 \quad -4 \quad -6 \\ \hline 1 \quad -2 \quad -3 \end{array}$$

$$(a-2)(a^2-2a-3)$$

$$(a-2)(a-3)(a+1)$$

Factor Theorem

③ a) $x^3 - x^2 - 14x + 24$ $(x-2)$ is a factor

$$(2)^3 - (2)^2 - 14(2) + 24$$

$$8 - 4 - 28 + 24$$

$$0$$

$$\begin{array}{r} x-2 \overline{) x^3 - x^2 - 14x + 24} \\ \underline{-(x^3 - 2x^2)} \\ x^2 - 14x \\ \underline{-(x^2 - 2x)} \\ -12x + 24 \\ \underline{-(-12x + 24)} \\ 0 \end{array}$$

Simple
Trinomial

$$(x-2)(x^2 + x - 12)$$

$$(x-2)(x-3)(x+4)$$

Synthetic Substitution

③ a) $x^3 - x^2 - 14x + 24$

$$(2)^3 - (2)^2 - 14(2) + 24$$

$$8 - 4 - 28 + 24$$

$$0$$

x value	coefficients			
$2 \downarrow$	1	-1	-14	24
		2	2	-24
	1	1	-12	

$$(x-2)(x^2 + x - 12)$$

$$(x-2)(x-3)(x+4)$$

$$\textcircled{33} \text{ a) } x^4 - 3x^3 - 5x^2 + 3x + 4 = 0$$

$$x=1 \quad 1 - 3 - 5 + 3 + 4 = 0$$

$$\begin{array}{r|rrrrr} 1 & 1 & -3 & -5 & 3 & 4 \\ & & 1 & -2 & -7 & -4 \\ \hline & 1 & -2 & -7 & -4 & \end{array}$$

$$(x-1)(x^3 - 2x^2 - 7x - 4)$$

-1 -2 +7 -4

Synthetic
Substitution

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -7 & -4 \\ & & -1 & 3 & 4 \\ \hline & 1 & -3 & -4 & \end{array}$$

$$(x-1)(x+1)(x^2 - 3x - 4)$$

Simple
Trinomial

$$\begin{array}{l} -4 \times 1 = -4 \\ -4 + 1 = -3 \end{array}$$

$$(x-1)(x+1)(x-4)(x+1)$$

$$(x-4)(x-1)(x+1)^2$$

③ e) $x^3 - 27x + 10 \div x - 5$

$$\begin{array}{r}
 \underline{\underline{x}} - 5 \overline{) x^3 + 0x^2 - 27x + 10} \\
 \underline{-(x^3 - 5x^2)} \\
 5x^2 - 27x \\
 \underline{-(5x^2 - 25x)} \\
 -2x + 10 \\
 \underline{-(-2x + 10)} \\
 0
 \end{array}$$

$$(x-5)(x^2+5x-2)$$

$$\textcircled{33} \quad x^4 - 3x^3 - 5x^2 + 3x + 4 = 0$$

$$(1)^4 - 3(1)^3 - 5(1)^2 + 3(1) + 4 = 0$$

$$1 - 3 - 5 + 3 + 4 = 0$$

$$0 = 0$$

$$\begin{array}{r|rrrrr} 1 & 1 & -3 & -5 & 3 & 4 \\ & & 1 & -2 & -7 & -4 \\ \hline & 1 & -2 & -7 & -4 & 0 \end{array}$$

$$(x-1)(x^3 - 2x^2 - 7x - 4) \leftarrow \text{Synthetic Sub.}$$

$$(-1)^3 - 2(-1)^2 - 7(-1) - 4$$

$$-1 - 2 + 7 - 4 = 0$$

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -7 & -4 \\ & & -1 & 3 & 4 \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

$$(x-1)(x+1)(x^2 - 3x - 4) \leftarrow \text{Simple Trinomial}$$

$$(x-1)(x+1)(x-4)(x+1)$$

$$\text{or } (x-4)(x-1)(x+1)^2$$

Advanced Math

31) c)

$$\begin{array}{r}
 \underline{\underline{x-3}} \overline{) x^4 + x^3 - 13x^2 - 7x + 30} \\
 \underline{-(x^4 - 3x^3)} \\
 4x^3 - 13x^2 \\
 \underline{-(4x^3 - 12x^2)} \\
 -x^2 - 7x + 30 \\
 \underline{-(-x^2 + 3x)} \\
 -10x + 30 \\
 \underline{-(-10x + 30)} \\
 0
 \end{array}$$

$(x-3)(x^3 + 4x^2 - x - 10)$ Factor further (Factor Theorem)

$(-2)^3 + 4(-2)^2 - (-2) - 10$
 $-8 + 16 + 2 - 10$
 0

$(x+2)$ is a factor

$$\begin{array}{r}
 \underline{\underline{x+2}} \overline{) x^3 + 4x^2 - x - 10} \\
 \underline{-(x^3 + 2x^2)} \\
 2x^2 - x - 10 \\
 \underline{-(2x^2 + 4x)} \\
 -5x - 10 \\
 \underline{-(-5x - 10)} \\
 0
 \end{array}$$

$(x-3)(x+2)(x^2 + 2x - 5)$

$(x-3)(x+2)(x^2 + 2x - 5) = x^4 + x^3 - 13x^2 - 7x + 30$

"factored form"

$$\begin{array}{r}
 \textcircled{31} \text{ f,} \\
 \underline{\underline{x-1}} \overline{3x^4 + 7x^3 + 5x^2 + 7x + 2} \\
 \underline{-(3x^5 - 3x^4)} \quad \downarrow \\
 7x^4 - 2x^3 \quad \downarrow \\
 \underline{-(7x^4 - 7x^3)} \quad \downarrow \\
 5x^3 + 2x^2 \quad \downarrow \\
 \underline{-(5x^3 - 5x^2)} \quad \downarrow \\
 7x^2 - 5x \quad \downarrow \\
 \underline{-(7x^2 - 7x)} \quad \downarrow \\
 2x - 2 \\
 \underline{-(2x - 2)} \\
 0
 \end{array}$$

$(x-1)(3x^4 + 7x^3 + 5x^2 + 7x + 2)$ Factor further "Factor Theorem"
 $3(-2)^4 + 7(-2)^3 + 5(-2)^2 + 7(-2) + 2$
 $48 - 56 + 20 - 14 + 2$
 0 $(x+2)$ is a factor

$$\begin{array}{r}
 \underline{\underline{x+2}} \overline{3x^3 + x^2 + 3x + 1} \\
 \underline{-(3x^4 + 6x^3)} \quad \downarrow \\
 x^3 + 5x^2 \quad \downarrow \\
 \underline{-(x^3 + 2x^2)} \quad \downarrow \\
 3x^2 + 7x \quad \downarrow \\
 \underline{-(3x^2 + 6x)} \quad \downarrow \\
 x + 2 \\
 \underline{-(x + 2)} \\
 0
 \end{array}$$

$(x-1)(x+2)(3x^3 + x^2 + 3x + 1)$ Group for a common factor

$$(x-1)(x+2)(x^2(3x+1) + 1(3x+1))$$

$$(x-1)(x+2)(3x+1)(x^2+1)$$

③ a) (i) $m^3 - 7m + 6 \div m - 1$

Synthetic Substitution

m value

coefficients

1

1 0 -7 6

1 1 -6

1 1 -6 \emptyset

"coefficients of other factor"

$(m-1)(m^2+m-6)$

$(m-1)(m-2)(m+3)$

Difference of Cubes

$$\left. \begin{array}{l} (a^3 - b^3) \rightarrow (a-b)(a^2 + ab + b^2) \\ (8x^3 - 125) \rightarrow (2x-5)(4x^2 + 10x + 25) \end{array} \right\}$$

Sum of Cubes

$$(a^3 + b^3) \rightarrow (a+b)(a^2 - ab + b^2)$$

Adv Math Quiz

- Factoring
- Composite Functions
- Sketching Functions
 - ↳ Roots (x_{int})
 - ↳ y_{int}
 - ↳ degree
 - ↳ stretch factor
 - ↳ local max/min

$$\textcircled{1} f) \quad x^4 + \underline{5x^2} + 49 \quad \sqrt{49} = 7 \cdot 2 = 14$$

$$(x^4 + 14x^2 + 49) - 9x^2$$

$$\boxed{(x^2 + 7)^2} - \boxed{9x^2}$$

$$(x^2 + 7 + 3x)(x^2 + 7 - 3x)$$

$$(x^2 + 3x + 7)(x^2 - 3x + 7)$$

$$\textcircled{1} \text{ e) } x^4 + 7x^2 + 12$$

$$(x^2 + 3)(x^2 + 4)$$

$$\text{f) } x^4 + \underline{5}x^2 + 49 \quad \sqrt{49} = 7 \cdot 2 = 14$$

$$(x^4 + 14x^2 + 49) - 9x^2$$

$$\boxed{(x^2 + 7)^2} - \boxed{9x^2}$$

$$((x^2 + 7) + 3x)((x^2 + 7) - 3x) \checkmark$$

$$(x^2 + 3x + 7)(x^2 - 3x + 7)$$

$$\textcircled{1} \text{ g) } \underline{49}x^2 - 84x + \underline{36}$$
$$(7x - 6)(7x - 6)$$
$$(7x - 6)^2$$

you could
use decomp.

Adv. Math

$$\begin{aligned} \textcircled{1} \text{ i) } & 9x^2 - 30x - a^2 + 25 \\ & (9x^2 - 30x + 25) - a^2 \\ & (3x - 5)(3x - 5) - a^2 \\ & \boxed{(3x - 5)^2} - \boxed{a^2} \\ & ((3x - 5) + a)((3x - 5) - a) \end{aligned}$$

$$\textcircled{a} \ a) \quad x^3 + 2x^2 - 11x - 12$$

$$(-1)^3 + 2(-1)^2 - 11(-1) - 12$$

$$-1 + 2 + 11 - 12$$

$$0$$

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -11 & -12 \\ & & -1 & -1 & 12 \\ \hline & 1 & 1 & -12 & \emptyset \end{array}$$

$$(x+1)(x^2 + x - 12)$$

$$(x+1)(x+4)(x-3)$$

Adv Math:

$$\textcircled{4} \quad f(x) = x^2 + 3 \quad g(x) = 2x - 3$$

a) $f(g(x))$

$$\begin{aligned} f(2x-3) &= (2x-3)^2 + 3 \\ &= 4x^2 - 12x + 9 + 3 \\ &= 4x^2 - 12x + 12 \end{aligned}$$

$$\textcircled{1} \text{ a) } 8x^3 - 125 \\ (2x - 5)(4x^2 + 10x + 25)$$

$$\text{h) } 5a^3 - 10a^2 - 40a \\ 5a(a^2 - 2a - 8) \\ (5a)(a - 4)(a + 2)$$

$$\begin{aligned}
 & \textcircled{f)} \quad x^4 + \underline{5}x^2 + 49 \qquad \sqrt{49} = 7 \cdot 2 = 14 \\
 & \quad (x^4 + 14x^2 + 49) - 9x^2 \\
 & \quad \boxed{(x^2 + 7)^2} - \boxed{9x^2} \\
 & \quad (x^2 + 7 + 3x)(x^2 + 7 - 3x)
 \end{aligned}$$

$$\textcircled{2} a) \cdot x^3 + 2x^2 - 11x - 12$$

$$(-1)^3 + 2(-1)^2 - 11(-1) - 12$$

$$-1 + 2 + 11 - 12$$

$$0$$

$$x = -1$$

$(x+1)$ is a factor

$$\begin{array}{r} \underline{-1} \Big| \quad 1 \quad 2 \quad -11 \quad -12 \\ \quad \quad \quad -1 \quad -1 \quad 12 \\ \hline \quad \quad 1 \quad 1 \quad -12 \end{array}$$

$$(x+1)(x^2 + x - 12)$$

$$(x+1)(x+4)(x-3)$$