Warm Up

Given the following matrices…

$$X = \begin{pmatrix} -1 & -2 \\ 3 & 1 \\ 2 & 5 \end{pmatrix} \qquad Y = \begin{pmatrix} 4 & -2 & 6 \\ 1 & 3 & 0 \end{pmatrix}$$

$$Z = \begin{pmatrix} 1 & -5 & 6 \\ -3 & 0 & 2 \\ 2 & -1 & 7 \end{pmatrix}$$

Determine the value of $2XY - 5Z$

Check your work on a TI-83 calculator when finished

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2*[A]*[B]-5[C]
  [[-17 17 -42]
   [41  -6 26 ]
   [16  27 -11]]
```

SPECIAL MATRICES

(1) Zero Matrix

- all entries are 0's.

(2) Identity Matrix - "I" (unit matrix)

Ex. $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

ex. 3x3
2x2

- square matrix with 1's along the diagonal and 0's everywhere else
- behaves like the number "1" in multiplication (any matrix multiplied by I is the same matrix)

2x2 2x2

$$\begin{pmatrix} 1 & -3 \\ 2 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ 2 & 1 \end{pmatrix}$$

(3) Inverses - two matrices whose product is a unit matrix are called inverses

i.e. A and B are inverses if $AB = I$ and $BA = I$

Ex. Find AB & BA

$$\begin{pmatrix} 1 & 4 \\ 2 & 9 \end{pmatrix} \times \begin{pmatrix} 9 & -4 \\ -2 & 1 \end{pmatrix} = \begin{bmatrix} 9+(-8) & -4+4 \\ 18+(-18) & -8+9 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2x2 2x2

A **B**

$$\begin{bmatrix} 9 & -4 \\ -2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 4 \\ 2 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Determinants

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ Det = ad - bc

Det = (product of major diagonal) - (product of other diagonal)

Ex. $\begin{pmatrix} 2 & 7 \\ 5 & 18 \end{pmatrix}$

$\text{Det} = 2(18) - (7)(5)$
$= 36 - 35$
$= 1$

$\begin{pmatrix} 12 & 10 \\ 7 & 6 \end{pmatrix}$

$\text{Det} = 12(6) - (10)(7)$
$= 72 - 70$
$= 2$

$\begin{pmatrix} -2 & -1 \\ 4 & 1 \end{pmatrix}$

$\text{Det} = (-2)(1) - (-1)(4)$
$= -2 - (-4)$
$= -2 + 4$
$= 2$

Finding an inverse of a 2 x 2 matrix using determinants...

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

1. Find the Determinant ($D = ad - bc$)
2. Form New Matrix
3. $\frac{1}{D} \times NewMatrix = Inverse$

Ex. $A = \begin{pmatrix} 0 & -3 \\ 4 & 5 \end{pmatrix}$ $A^{-1} = \begin{bmatrix} 5/12 & 1/4 \\ -1/3 & 0 \end{bmatrix}$ ← Inverse

① Det = $0(5) - (-3)(4)$
$= 0 - (-12)$
$= 12$

② New Matrix
$\begin{bmatrix} 5 & 3 \\ -4 & 0 \end{bmatrix}$

③ $A^{-1} = \frac{1}{12}\begin{bmatrix} 5 & 3 \\ -4 & 0 \end{bmatrix}$
$= \begin{bmatrix} 5/12 & 3/12 \\ -4/12 & 0 \end{bmatrix}$
$= \begin{bmatrix} 5/12 & 1/4 \\ -1/3 & 0 \end{bmatrix}$

Find the inverse

$$\begin{pmatrix} 12 & 10 \\ 7 & 6 \end{pmatrix}$$

① D = $12(6) - (10)(7)$
$= 72 - 70$
$= 2$

② New Matrix:
$\begin{bmatrix} 6 & -10 \\ -7 & 12 \end{bmatrix}$

③ Inv = $\frac{1}{2}\begin{bmatrix} 6 & -10 \\ -7 & 12 \end{bmatrix}$
$= \begin{bmatrix} 3 & -5 \\ -7/2 & 6 \end{bmatrix}$

Finding the Inverse using the Identity Matrix

Ex. $\left(\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 2 & 9 & 0 & 1 \end{array}\right)$ ①$-4$② ②①$-$③

Perform elementary operations to make left side unit matrix

$\left[\begin{array}{cc|cc} 1 & 0 & 9 & -4 \\ 0 & -1 & 2 & -1 \end{array}\right]$ $-1$③

$\left[\begin{array}{cc|cc} 1 & 0 & 9 & -4 \\ 0 & 1 & -2 & 1 \end{array}\right]$ ← Inverse

Homework

#1 using determinants

#2

#3 using identity matrix