Warm Up

Solve the following system of equations and identify the type of system... Inconsistant

$$3x + 2y + z = 3$$
$$x - 3y + z = 4$$
$$-6x - 4y - 2z = 1$$

1 + 3

$6x + 4y + 2z = 6$

$(+)\ -6x - 4y - 2z = 1$

$0 \neq 7$

No Solution

Questions from homework

① Determinant Method

③ g) $\begin{bmatrix} 6 & -3 \\ -2 & 5 \end{bmatrix}$

① Det = $ad - bc$
$= (6)(5) - (-3)(-2)$
$= 30 - 6$
$= 24$

② New Matrix: $\begin{bmatrix} 5 & 3 \\ 2 & 6 \end{bmatrix}$

③ $\frac{1}{24}\begin{bmatrix} 5 & 3 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} \frac{5}{24} & \frac{1}{8} \\ \frac{1}{12} & \frac{1}{4} \end{bmatrix}$

② Unit Matrix Method

③ g) $\left[\begin{array}{cc|cc} 6 & -3 & 1 & 0 \\ -2 & 5 & 0 & 1 \end{array}\right]$ $5R1 + 3R2$, $3R2 + R1$

$\left[\begin{array}{cc|cc} 24 & 0 & 5 & 3 \\ 0 & 12 & 1 & 3 \end{array}\right]$ $R1 \div 24$, $R2 \div 12$

$\left[\begin{array}{cc|cc} 1 & 0 & 5/24 & 1/8 \\ 0 & 1 & 1/12 & 1/4 \end{array}\right]$

Solving Equations Using Matrices

or 3X3

Matrix Elimination involves taking the coefficients from a 2X2 system, placing them in a matrix, and working to make a new matrix by multiplying, dividing, and combining rows.

The combination of the coefficients from a system of equations and their solutions in an equivalent form is called an **augmented matrix**.

Ex. $2x + y + 3z = 0$
$x + y - 2z = -1$
$x - 2y - z = 3$

$$\longrightarrow \left(\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 1 & 1 & -2 & -1 \\ 1 & -2 & -1 & 3 \end{array}\right)$$

3x4

$x + 3y = 4$
$3x + 4y = 2$

$$\longrightarrow \left(\begin{array}{cc|c} 1 & 3 & 4 \\ 3 & 4 & 2 \end{array}\right)$$

2x3

Row Reduced Echelon Form

The goal in solving a system of equation using matrices is to obtain a new matrix - row reduced echelon form of a matrix. It takes the form:

$$\begin{pmatrix} 1 & 0 & x \\ 0 & 1 & y \end{pmatrix} \quad \textbf{Or} \quad \begin{pmatrix} 1 & \# & \# & x \\ 0 & 1 & \# & y \\ 0 & 0 & 1 & z \end{pmatrix}$$

To reduce a matrix to its row echelon form, we can:
a) Multiply or divide a row by a constant.
b) Add or subtract one row from another.
c) Interchange rows.

Solve the following system of equations using an augmented matrix reduced to its row echelon form...

$$x + 3y = 4$$

$$3x + 4y = 2$$

$$\left[\begin{array}{cc|c} 1 & 3 & 4 \\ 3 & 4 & 2 \end{array}\right] \quad 4(R1) - 3(R2)$$

$$(R2) - 3(R1)$$

$$\left[\begin{array}{cc|c} -5 & 0 & 10 \\ 0 & -5 & -10 \end{array}\right] \quad (R1) \div -5$$

$$(R2) \div -5$$

$$\left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 2 \end{array}\right]$$

$$x = -2$$

$$y = 2$$

Try this one on your own...

$$3x + 2y = 12$$
$$2x + 3y = 13$$

1. **Express system in the form of an augmented matrix**
2. **Eliminate *"x"* in equation 2 and 3.**
3. **Eliminate *"y"* in equation 3 (must add/subtract 2 and 3)**
4. **Create triangle of zeroes and solve.**

Ex. $2x + y - z = -1$
$3x - y + 2z = 8$
$2x + 2y - 3z = -6$

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & -1 \\ 3 & -1 & 2 & 8 \\ 2 & 2 & -3 & -6 \end{array}\right] \begin{array}{l} 2R2 - 3R1 \\ R3 - R1 \end{array} \left[\begin{array}{ccc|c} 2 & 1 & -1 & -1 \\ 0 & -5 & 7 & 19 \\ 0 & 1 & -2 & -5 \end{array}\right] 5R3 + R2 \left[\begin{array}{ccc|c} 2 & 1 & -1 & -1 \\ 0 & -5 & 7 & 19 \\ 0 & 0 & -3 & -6 \end{array}\right]$$

$-3z = -6$
$-3z = -6$
$z = 2$

$-5y + 7z = 19$
$-5y + 7(2) = 19$
$-5y + 14 = 19$
$-5y = 5$
$y = -1$

$2x + y - z = -1$
$2x + (-1) - (2) = -1$
$2x - 3 = -1$
$2x = 2$
$x = 1$

$(1, -1, 2)$

Homework