Solving Polynomial Inequalities

Using the Graph

where does the function have positive "y" values

A polynomial inequality, $x^3 + x^2 - 9x - 9 > 0$, can be solved by examining the graph of the corresponding polynomial function, $y = (x^3 + x^2)(-9x - 9)$

• Degree → 3rd

• Stretch Factor: $a = 1$

• Roots: $y = 0$

$y = x^2(x+1) - 9(x+1)$

$y = (x+1)(x^2-9)$

$y = (x+1)(x+3)(x-3)$

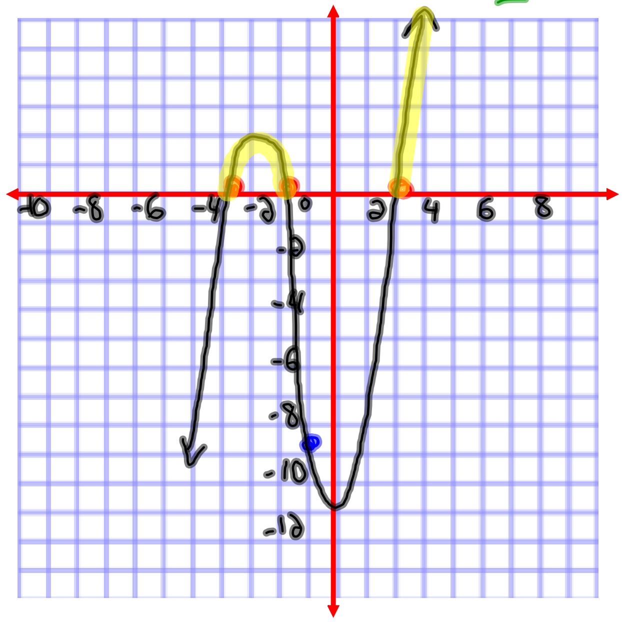
$\boxed{x = -3, -1, 3}$

• y intercept $(x=0)$

$y = x^3 + x^2 - 9x - 9$

$y = (0)^3 + (0)^2 - 9(0) - 9$

$\boxed{y = -9}$



$\boxed{-3 < x < -1 \text{ and } x > 3}$

$x \in (-3,-1)$ $x \in (3,\infty)$

$\boxed{x \in (-3,-1) \cup (3,\infty)}$

Interval Notation

The statement *-2* < ***x*** < *3* can be written as ***x*** $\in$ ***(-2, 3)***; that is *x* belongs to the interval ***(-2, 3).*** The round brackets mean that *x* is not equal to ***-2 or 3.***

The statement -4 ≤ ***x*** ≤ *2* can be written as ***x*** $\in$ **[*-4, 2*].** The square brackets mean that *x* may be equal to ***-4 or 2.***

Explain the meaning of the following interval notations.

$x \in (-\infty, 2)$	$-\infty < x < 2$	$x < 2$
$x \in (-\infty, 2]$	$-\infty < x \leq 2$	$x \leq 2$
$x \in (3, \infty)$	$3 < x < \infty$	$x > 3$
$x \in [3, \infty)$	$3 \leq x < \infty$	$x \geq 3$

Note: Infinity cannot be inclusive

Solving Polynomial Inequalities

Using the Number Line

Example: $x^3 + x^2 > 6x$

Step 1: State the Roots of the function

Step 2: Draw a number line and mark the roots of the equation. These roots separate the rest of the number line into three intervals.

$x \in (-\infty, \text{small x-int})$

$x \in (\text{small x-int}, \text{large x-int})$

$x \in (\text{large x-int}, \infty)$

Step 3: The value of the expression $x^3 + x^2 - 6x$ has the same sign throughout each interval in step 2 **because a function can only change signs at a root.** Therefore, choose a *test value of x* in each interval and evaluate the expression. Write a *plus or a minus* over that interval on the number line to indicate whether the expression is positive or negative.

Step 4: State the intervals for which $x^3 + x^2 - 6x > 0$

Using the Number Line

Example: $x^3 + x^2 > 6x$

$$\boxed{x^3 + x^2 - 6x > 0}$$

'+' y values

Step 1: State the Roots of the function

$y = x^3 + x^2 - 6x$

$y = x(x^2 + x - 6)$

$y = x(x+3)(x-2)$

Roots:

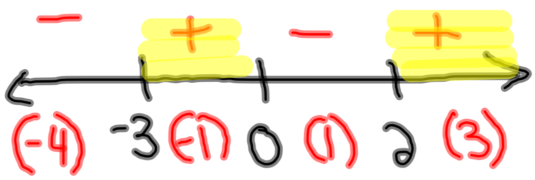
$$\boxed{x = -3, 0, 2}$$

Step 2: Draw a number line and mark the roots of the equation. These roots separate the rest of the number line into three intervals.

x $\in$ ***(-∞ , small x-int)***

x $\in$ ***(small x-int, large x-int)***

x $\in$ ***(large x-int, ∞)***



Step 3: The value of the expression $x^3 + x^2 - 6x$ has the same sign throughout each interval in step 2 **because a function can only change signs at a root.** Therefore, choose a *test value of* x in each interval and evaluate the expression. Write a *plus or a minus* over that interval on the number line to indicate whether the expression is positive or negative.

$x = -4$

$y = x(x+3)(x-2)$

$y = (-4)(-1)(-6)$

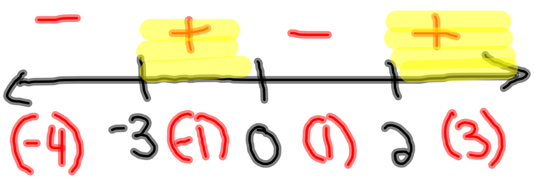
$y = -24$

$x = -1$

$y = (-)(+)(-)$

$y = +$

Step 4: State the intervals for which $x^3 + x^2 - 6x > 0$



$x \in (-3, 0) \cup (2, \infty)$

Homework