

Warm Up

Solve the following system of equations:

$$\begin{aligned}4x + 9 &= 3y - 6z \\3z &= 10 + 2x + 4y \\2y &= 4z - 11 - 3x\end{aligned}$$

$$\begin{aligned}4x - 3y + 6z &= -9 \\-2x - 4y + 3z &= 10 \\3x + 2y - 4z &= -11\end{aligned}$$

$$\begin{array}{l}4x - 3y + 6z = -9 \\ \textcircled{A} \quad -4x - 8y + 6z = 20 \\ \hline -11y + 12z = 11\end{array} \quad \begin{array}{l}-6x - 12y + 9z = 30 \\ \textcircled{+} \quad 6x + 4y - 8z = -22 \\ \hline -8y + z = 8\end{array} \quad \text{"2x2 system"}$$

$$\begin{aligned}-11y + 12z &= 11 \\ \Leftrightarrow \frac{-96y + 12z}{85y} &= \frac{96}{-85} \\ 85y &= -85 \\ \boxed{y} &= \boxed{-1}\end{aligned}$$

$$\begin{aligned}-8y + z &= 8 \\ -8(-1) + z &= 8 \\ 8 + z &= 8 \\ \boxed{z} &= \boxed{0}\end{aligned}$$

$$\begin{aligned}4x - 3y + 6z &= -9 \\ 4x - 3(-1) + 6(0) &= -9 \\ 4x + 3 + 0 &= -9 \\ 4x &= -12 \\ \boxed{x} &= \boxed{-3}\end{aligned}$$

$$(-3, -1, 0)$$

Questions from Homework

②e) $3x - 4y + 5z = 26$ $6x - 8y + 10z = 52$ $6x - 2y - 3z = -39$
 $6x - 2y - 3z = -39$ $\Leftrightarrow 6x - 2y - 3z = -39$ $\Leftrightarrow 6x + 18y - 24z = -186$
 $x + 3y - 4z = -31$ $-6y + 13z = 91$ $-20y + 21z = 147$

$-60y + 130z = 910$
 $\Leftrightarrow \frac{-60y + 130z = 910}{-60y + 63z = 441}$
 $67z = 469$
 $z = 7$

$-6y + 13z = 91$
 $-6y + 13(7) = 91$
 $-6y + 91 = 91$
 $-6y = 0$
 $y = 0$

$x + 3y - 4z = -31$
 $x + 3(0) - 4(7) = -31$
 $x + 0 - 28 = -31$
 $x = -3$

$(-3, 0, 7)$

③c) $4x - 3y + 8z = 5$ $8x - 6y + 16z = 10$ $6x + 9y + 16z = 4$
 $6x + 9y + 16z = 4$ $\Leftrightarrow 6x + 9y + 16z = 4$ $\Leftrightarrow 24x - 96y - 16z = 40$
 $3x - 12y - 2z = 5$ $2x - 15y = 6$ $30x - 87y = 44$

$30x - 225y = 90$
 $\Leftrightarrow \frac{30x - 225y = 90}{30x - 87y = 44}$
 $-138y = 46$
 $y = \frac{46}{-138}$
 $y = -\frac{1}{3}$

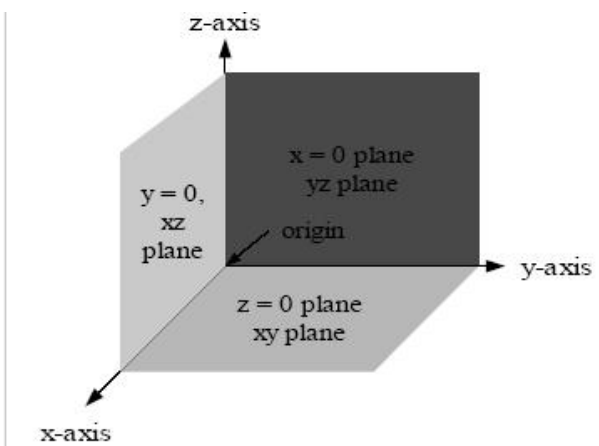
$2x - 15(\frac{1}{3}) = 6$
 $2x + 5 = 6$
 $2x = 1$
 $x = \frac{1}{2}$

$4(\frac{1}{2}) - 3(\frac{1}{3}) + 8z = 5$
 $2 + 1 + 8z = 5$
 $8z = 2$
 $z = \frac{1}{4}$

$(\frac{1}{2}, -\frac{1}{3}, \frac{1}{4})$

ALGEBRA OF 3-SPACE

- Coordinate geometry that represents space in **three** dimensions
- Coordinates are in the form of an ordered triplet **(x, y, z)**
- Three planes exist: **xy** plane, **xz** plane, **yz** plane

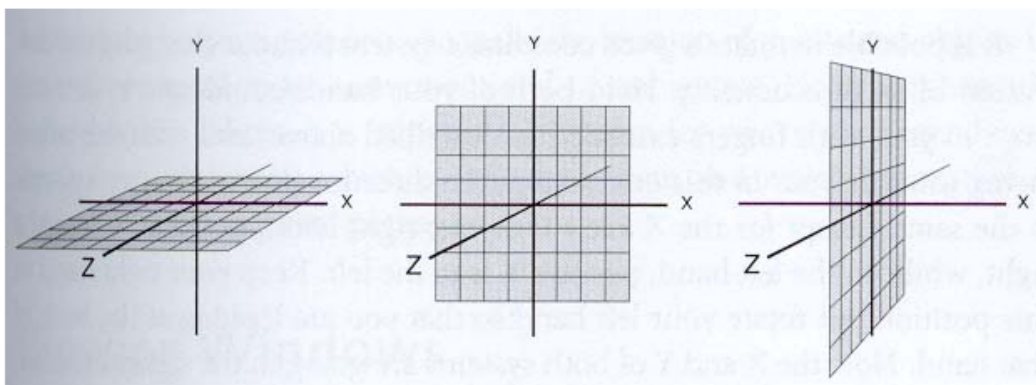


x axis -axis coming "out of the page"
y axis - horizontal axis
z axis - vertical axis

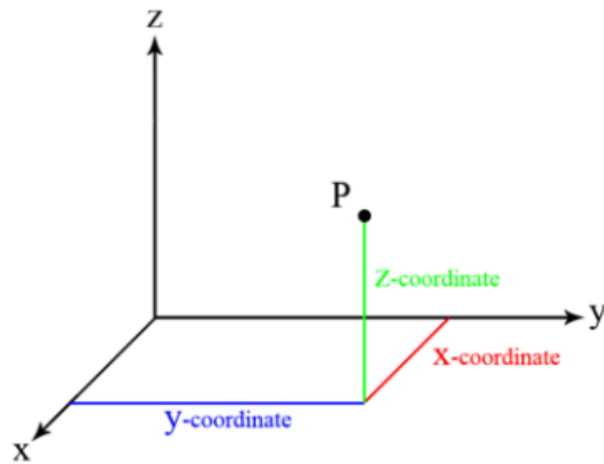
xz plane

xy plane

yz plane



Plotting Points in 3-Space

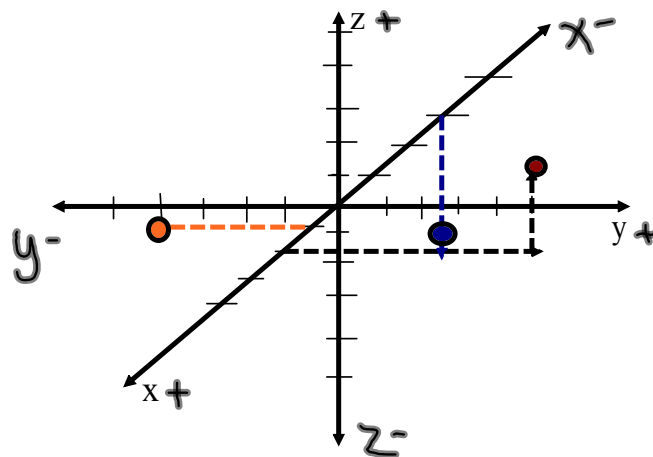


Plotting points in 3-space...

Ex: a) $(2, 6, 3)$

b) $(-3, 0, -4)$

c) $(1, -4, 0)$

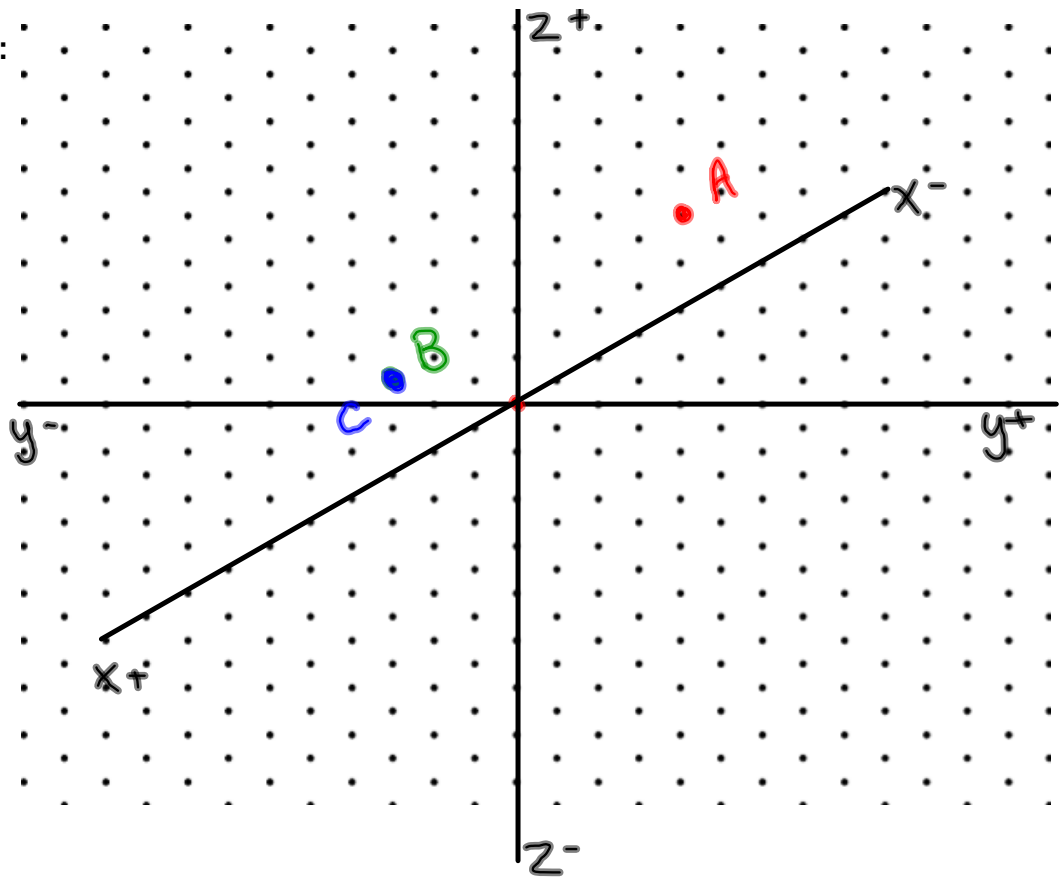


Plot the following:

A (-2, 1, 3) ●

B (3, 0, 2) ●

C (-1, -2, 0) ●



Finding Intercepts in 3D

As in two dimensions...

x intercept can be found when $y = 0$ and $z = 0$

$$(x,y,z) \longrightarrow (x,y,0)$$

y intercept can be found when $x = 0$ and $z = 0$

$$(x,y,z) \longrightarrow (0,y,0)$$

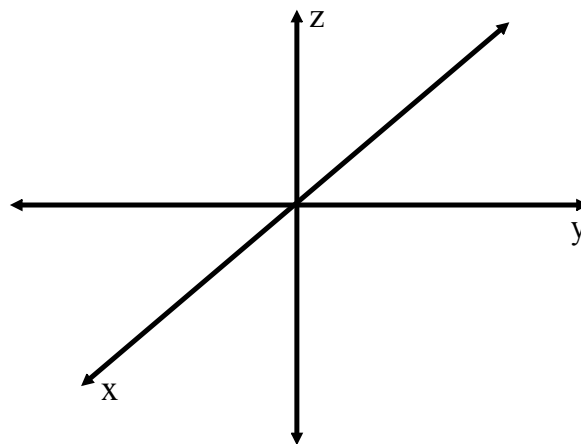
z intercept can be found when $x = 0$ and $y = 0$

$$(x,y,z) \longrightarrow (0,0,z)$$

Plotting Planes in 3-Space

- Use the **intercept method** to plot the x, y, and z intercepts to form a triangle
- The triangle is part of the plane being sketched

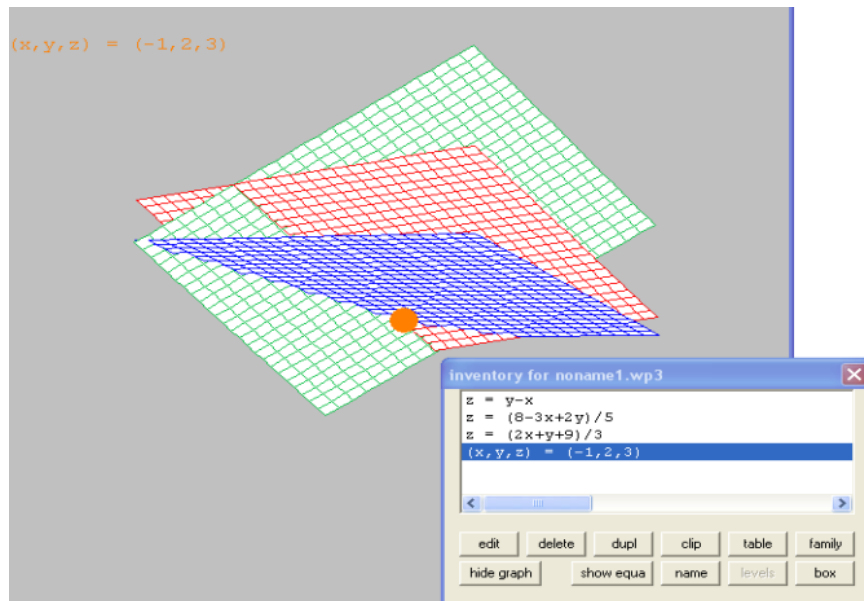
Ex. $2x - y + 3z = 6$



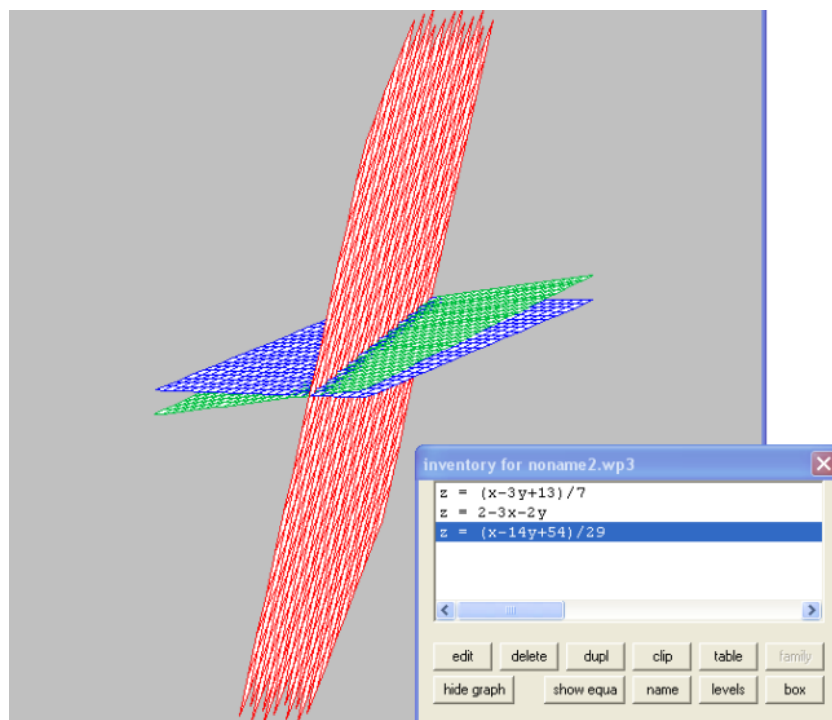
Types of Systems

Remember: Looking at **intersecting planes!**

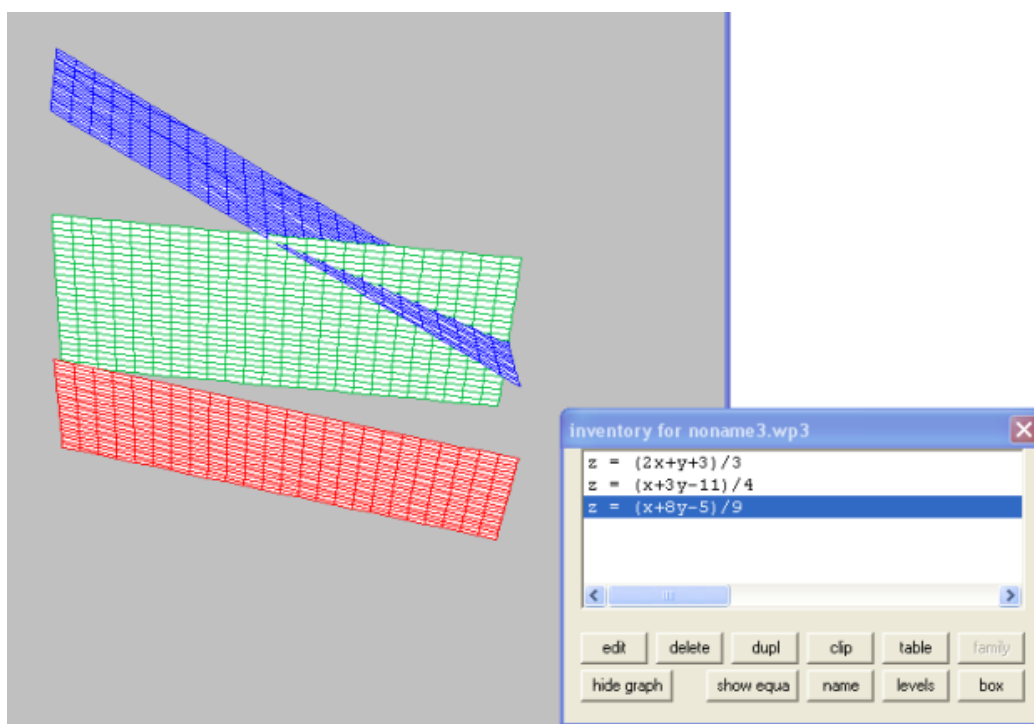
Consistent: At least one solution



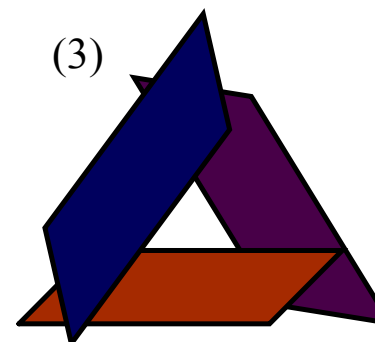
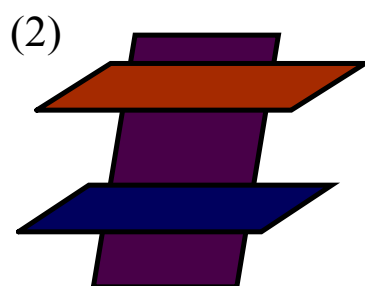
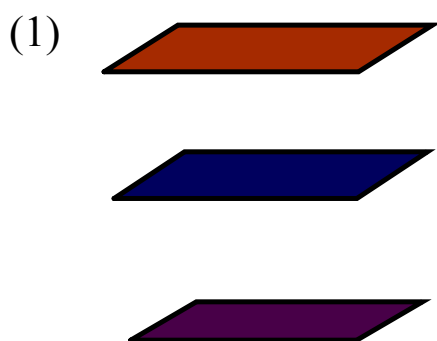
Dependent: Infinite number of solutions



Inconsistent: No Solutions



3 Possible Orientations That Give No Solution...



I. Consistent System with a Unique Solution

$$x - y + z = 0$$

$$3x - 2y + 5z = 8$$

$$2x + y - 3z = -9$$

STEPS:

- 1) Eliminate one of the variables
- 2) Solve the 2 x 2 system
- 3) Use "backward substitution" to obtain a solution

Homework

Handout: Solving Systems of Equations in 3-Space

#

II. Consistent System with a Dependent Solution (must create a parametric solution)

$$x - 3y - 7z = -13$$

$$3x + 2y + z = 2$$

$$x - 14y - 29z = -54$$

Write a general solution in terms of a parameter (i.e. $z = t$). For each value assigned to the parameter there will be one distinct solution.