Questions from Homework

6 b) $y = 2x^2 - 6\sqrt{x}$ at $(4, 20)$

$\uparrow \quad \uparrow$
$x_1 \quad y_1$

① Differentiate:

$y = 2x^2 - 6x^{\frac{1}{2}}$

$y' = 4x - 3x^{-\frac{1}{2}}$

$y' = 4x - \dfrac{3}{x^{\frac{1}{2}}}$

$y' = 4x - \dfrac{3}{\sqrt{x}}$

② Sub in x-value

$y' = 4(4) - \dfrac{3}{\sqrt{(4)}}$

$y' = 16 - \dfrac{3}{2}$

$y' = \dfrac{32}{2} - \dfrac{3}{2} = \boxed{\dfrac{29}{2}}$ ← Slope of tangent "m"

③ Find equation:

$y - y_1 = m(x - x_1)$

$y - 20 = \dfrac{29}{2}(x - 4)$

$y - 20 = \dfrac{29x}{2} - \dfrac{116}{2}$

$2 \cdot y - 20 \stackrel{2}{=} \dfrac{29x}{2} \stackrel{2}{-} 58$

$2y - 40 = 29x - 116$

$\boxed{0 = 29x - 2y - 76}$

7 c) $g(x) = 4x^3 - \dfrac{6}{x^2} + 14x$

$g(x) = 4x^3 - 6x^{-2} + 14x$

$g'(x) = 12x^2 + 12x^{-3} + 14x^0$

$g'(x) = 12x^2 + \dfrac{12}{x^3} + 14$

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Warm Up

Differentiate the following:

$$f(x) = -4x^2 - 5x(x^3 + 7)^2 + 2\sqrt[5]{x^9} - \frac{5}{x^{10}} + \frac{7x^2}{\sqrt{x}}$$

$$f(x) = -4x^2 - 5x\left(x^6 + 14x^3 + 49\right) + 2x^{9/5} - 5x^{-10} + 7x^2\left(x^{-1/2}\right)$$

$$f(x) = -4x^2 - 5x^7 - 70x^4 - 245x + 2x^{9/5} - 5x^{-10} + 7x^{3/2}$$

$$f'(x) = -8x - 35x^6 - 280x^3 - 245 + \frac{18}{5}x^{4/5} + 50x^{-11} + \frac{21}{2}x^{1/3}$$

$$f'(x) = -8x - 35x^6 - 280x^3 - 245 + \frac{18}{5}x^{4/5} + \frac{50}{x^{11}} + \frac{21}{2}x^{1/2}$$

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Differentiation Rules

Product Rule:

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

Express the product rule verbally if you are considering a function of the form...

$$f(x) = (First) \times (Second)$$

In words, **the Product Rule** says that the **derivative of a product of two functions is:** **the first function times the derivative of the second function, plus the derivative of the first function times the second function**

$$(fg)' = fg' + f'g$$

Get in the habit of verbalizing the rule as you differentiate...it will help when the functions get more complicated.

Examples:

$$y = (2x^3 + 5)(3x^2 - x)$$

$$y' = (2x^3 + 5)(6x-1) + 6x^2(3x^2-x)$$

$$y' = 12x^4 - 2x^3 + 30x - 5 + 18x^4 - 6x^3$$

$$\boxed{y' = 30x^4 - 8x^3 + 30x - 5}$$

$f(x) = 2x^3 + 5$

$f'(x) = 6x^2$

$g(x) = 3x^2 - x$

$g'(x) = 6x - 1$

$$f(x) = \sqrt{x}(2 - 3x) = (x^{1/2})(2 - 3x)$$

$$f'(x) = x^{1/2}(-3) + \frac{1}{2}x^{-1/2}(2 - 3x)$$

$$f'(x) = -3x^{1/2} + x^{-1/2} - \frac{3}{2}x^{1/2}$$

$$f'(x) = \frac{1}{x^{1/2}} - \frac{9}{2}x^{1/2}$$

$$f'(x) = \frac{1}{\sqrt{x}} - \frac{9\sqrt{x}}{2}$$

$$\boxed{f'(x) = \frac{2 - 9x}{2\sqrt{x}}}$$

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Examples:

$$f(x) = \left(7x^3 - x^2 + 5\right)\left(x^9 + 3x - 5\right)$$

$$f'(x) = \left(7x^3 - x^2 + 5\right)\left(9x^8 + 3\right) + \left(21x^2 - 2x\right)\left(x^9 + 3x - 5\right)$$

$$f'(x) = 63x^{11} + 21x^3 - 9x^{10} - 3x^2 + 45x^8 + 15 + 21x^{11} + 63x^3 - 105x^2 - 2x^{10} - 6x^2 + 10x$$

$$f'(x) = 84x^{11} - 11x^{10} + 45x^8 + 84x^3 - 114x^2 + 10x + 15$$

$$h(t) = \left(t^3 - 5t\right)\left(6\sqrt{t} - t^{-5}\right)$$

Homework