Chain Rule:

4. $G(x) = \sqrt{x^4 - x + 1} = (x^4 - x + 1)^{1/2}$

$G'(x) = \frac{1}{2}(x^4 - x + 1)^{-1/2}(4x^3 - 1)$

$G'(x) = \frac{1}{2(x^4 - x + 1)^{1/2}} \cdot (4x^3 - 1)$

$G'(x) = \frac{4x^3 - 1}{2\sqrt{x^4 - x + 1}}$

8. $y = \frac{4}{\sqrt{9 - x^2}} = \frac{4}{(9 - x^2)^{1/2}} = 4(9 - x^2)^{-1/2}$

$y' = -2(9 - x^2)^{-3/2}(-2x)$

$y' = 4x(9 - x^2)^{-3/2}$

$y' = \frac{4x}{(9 - x^2)^{3/2}} = \frac{4x}{\sqrt{(9 - x^2)^3}}$

10. $y = \sqrt{x + \sqrt{x}} = (x + \sqrt{x})^{1/2} = (x + x^{1/2})^{1/2}$

$y' = \frac{1}{2}(x + x^{1/2})^{-1/2}\left(1 + \frac{1}{2}x^{-1/2}\right)$

$y' = \left[\frac{1}{2(x + \sqrt{x})^{1/2}}\right]\left[\frac{1}{1} + \frac{1}{2\sqrt{x}}\right]$ Add by finding common denom

$y' = \left[\frac{1}{2\sqrt{x + \sqrt{x}}}\right]\left[\frac{2\sqrt{x} + 1}{2\sqrt{x}}\right]$

$y' = \frac{2\sqrt{x} + 1}{4\sqrt{x}\sqrt{x + \sqrt{x}}}$ or $\frac{2\sqrt{x} + 1}{4\sqrt{x^2 + x^{3/2}}}$

⑦ $y=\frac{1}{(x^3+2x^2+1)^2}=1(x^3+2x^2+1)^{-2}$

$y'=-2(x^3+2x^2+1)^{-3}(3x^2+4x)$

$y'=\frac{-2(3x^2+4x)}{(x^3+2x^2+1)^3}$

⑧ $y=\frac{4}{\sqrt{9-x^2}}=\frac{4}{(9-x^2)^{1/2}}=4(9-x^2)^{-1/2}$

$y'=-2(9-x^2)^{-3/2}(-2x)$

$y'=\frac{4x}{(9-x^2)^{3/2}}=\frac{4x}{\sqrt{(9-x^2)^3}}$

⑩ $y=\sqrt{x+\sqrt{x}}=(x+\sqrt{x})^{1/2}=(x+x^{1/2})^{1/2}$

$y'=\frac{1}{2}(x+x^{1/2})^{-1/2}\left(1+\frac{1}{2}x^{-1/2}\right)$

$y'=\frac{1}{2}(x+\sqrt{x})^{-1/2}\left(\frac{1}{1}+\frac{1}{2\sqrt{x}}\right)$

add fractions
CD: $2\sqrt{x}$

$y'=\left[\frac{1}{2\sqrt{x+\sqrt{x}}}\right]\left[\frac{2\sqrt{x}}{2\sqrt{x}}+\frac{1}{2\sqrt{x}}\right]$

$y'=\left[\frac{1}{2\sqrt{x+\sqrt{x}}}\right]\left[\frac{2\sqrt{x}+1}{2\sqrt{x}}\right]$

$y'=\frac{2\sqrt{x}+1}{4\sqrt{x}\sqrt{x+\sqrt{x}}}$

Limits

① a) $\lim\limits_{x\to 0} \dfrac{\cancel{(x+2)}\dfrac{2}{\cancel{x+2}} - \dfrac{1}{1}(x+2)}{x(x+2)}$

$\lim\limits_{x\to 0} \dfrac{2-(x+2)}{x(x+2)}$

$\lim\limits_{x\to 0} \dfrac{\cancel{2}-x-\cancel{2}}{x(x+2)}$

$\lim\limits_{x\to 0} \dfrac{-\cancel{x}^{1}}{{}_{1}\cancel{x}(x+2)} = \boxed{-\dfrac{1}{2}}$

b) $\lim\limits_{x\to \infty} \dfrac{(2-3x^2)^2}{6x^4-7x^2-5}$

$\lim\limits_{x\to \infty} \dfrac{4-12x^2\boxed{+9}x^4}{\boxed{6}x^4-7x^2-5} = \dfrac{9}{6} = \boxed{\dfrac{3}{2}}$

Review:

④ $g(x) = (x^2 - 3x + 4)(2x^2 + 4x)$

$g'(x) = (x^2 - 3x + 4)(4x + 4) + (2x - 3)(2x^2 + 4x)$

$g'(x) = 4x^3 + 4x^2 - 12x^2 - 12x + 16x + 16 + 4x^3 + 8x^2 - 6x^2 - 12x$

$g'(x) = 8x^3 - 6x^2 - 8x + 16$

Differentiation Rules:

⑥ $y=(1+x)^{10}$ $x=\underline{0}$ $y=\underline{1}$ $(0,1)$

* $y=(1+0)^{10}=\underline{1}$

① Find Derivative

$y'=10(1+x)^9(1)$

$y'=10(1+x)^9$

② Find Slope $(x=0)$

$y'(0)=10(1+0)^9$

$=10(1)^9$

$\boxed{=10}$ ← m

③ Equation

$y-y_1=m(x-x_1)$

$y-1=10(x-0)$

$y-1=10x$

$\boxed{0=10x-y+1}$

③ Sum & Difference Rule:

a) $y=x^{10}+20x^5-12\sqrt[4]{x^3}+30$

$=x^{10}+20x^5-12x^{3/4}+30$

$y'=10x^9+100x^4-9x^{-1/4}+0$

$=10x^9+100x^4-\frac{9}{x^{1/4}}$

Limits

① a) $$\lim_{x\to 0} \frac{\cancel{(x+2)}\frac{2}{\cancel{x+2}} - 1\,(x+2)}{x\,(x+2)}$$

$$\lim_{x\to 0} \frac{2-x-2}{x(x+2)}$$

$$\lim_{x\to 0} \frac{\cancel{-x}}{\cancel{x}(x+2)} = -\frac{1}{2}$$

Piecewise Functions:

$>, <$ → open dot

$\geq, \leq, =$ → closed dot

④ $$\boxed{f'(x) = \lim_{h\to 0} \frac{\boxed{f(x+h)} - \boxed{f(x)}}{h}}$$

b) $f(x) = \boxed{\dfrac{2x-1}{4x}}$

$$f(x+h) = \frac{2(x+h)-1}{4(x+h)} = \boxed{\frac{2x+2h-1}{4x+4h}}$$

$$f'(x) = \lim_{h\to 0} \frac{\frac{2x+2h-1}{(4x+4h)} - \left(\frac{2x-1}{(4x)}\right)}{h}$$

$$= \lim_{h\to 0} \frac{4x(2x+2h-1) - (2x-1)(4x+4h)}{h(4x)(4x+4h)}$$

$$= \lim_{h\to 0} \frac{8x^2+8xh-4x - (8x^2+8xh-4x-4h)}{h(4x)(4x+4h)}$$

$$= \lim_{h\to 0} \frac{\cancel{8x^2}+\cancel{8xh}\cancel{-4x}\cancel{-8x^2}\cancel{-8xh}\cancel{+4x}+4h}{h(4x)(4x+4h)}$$

$$= \lim_{h\to 0} \frac{4\cancel{h}}{\cancel{h}(4x)(4x+4h)} = \frac{4}{(4x)^2} = \frac{4}{16x^2} = \boxed{\frac{1}{4x^2}}$$

Chain Rule:

18 $y=\sqrt{x+\sqrt{x}} = (x+\sqrt{x})^{1/2} = (x+x^{1/2})^{1/2}$

$y' = \frac{1}{2}(x+x^{1/2})^{-1/2}(1+\frac{1}{2}x^{-1/2})$

$y' = \left[\frac{1}{2(x+x^{1/2})^{1/2}}\right]\left[\frac{1}{1}+\frac{1}{2x^{1/2}}\right]$

$y' = \left[\frac{1}{2\sqrt{x+\sqrt{x}}}\right]\left[\frac{2\sqrt{x}+1}{2\sqrt{x}}\right]$

$y' = \frac{2\sqrt{x}+1}{4\sqrt{x}\sqrt{x+\sqrt{x}}}$ or $\frac{2x^{1/2}+1}{4x^{1/2}(x+x^{1/2})^{1/2}}$

$$\lim_{x\to\infty} \frac{1-9x^2}{3x^2+2x+1} = \frac{-9}{3} = \boxed{-3}$$

$$\lim_{x\to\infty} \frac{1-9x^2}{3x+1} = \boxed{\text{DNE}}$$

$$\lim_{x\to\infty} \frac{3x+1}{1-9x^2} = \boxed{0}$$

Ex 2.5

4

$f(2) = \boxed{3}$

$f'(2) = \boxed{5}$

$g(2) = \boxed{-1}$

$g'(2) = \boxed{-4}$

$$\left(\frac{f}{g}\right)'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2}$$

$$= \frac{(-1)(5) - (3)(-4)}{(-1)^2}$$

$$= \frac{-5 + 12}{1}$$

$$= 7$$

Ex: 2.5

6) $y = \frac{x^2}{2x+5}$

$y' = \frac{(2x+5)(2x) - x^2(2)}{(2x+5)^2}$

$y' = \frac{4x^2 + 10x - 2x^2}{(2x+5)^2}$

$y' = \frac{2x^2+10x}{(2x+5)^2} = \frac{2x(x+5)}{(2x+5)^2}$

$\frac{0}{1} = \frac{2x(x+5)}{(2x+5)^2}$

$(2x)(x+5) = 0$

$2x = 0$ | $x+5=0$

$x = 0$ | $x = -5$

$x = 0$

$y = \frac{(0)^2}{2(0)+5}$

$= \frac{0}{5}$

$= 0$

$(0,0)$

$x = -5$

$y = \frac{(-5)^2}{2(-5)+5}$

$= \frac{25}{-5}$

$= -5$

$(-5,-5)$