Questions From Homework

② Let x = 1st number
Let y = 2nd number

$x + y = 8$

$x = \boxed{8 - y}$

$x = 8 - 2$

$x = 6$

$S = x^2 + y^3$

$S = (8-y)^2 + y^3$

$S = 64 - 16y + y^2 + y^3$

$S' = -16 + 2y + 3y^2$ decomp.

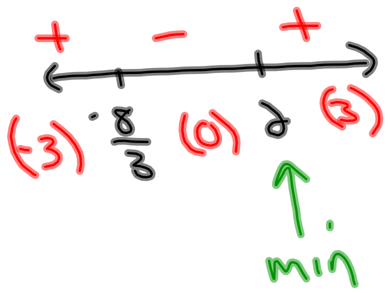
$S' = 3y^2 + 2y - 16$

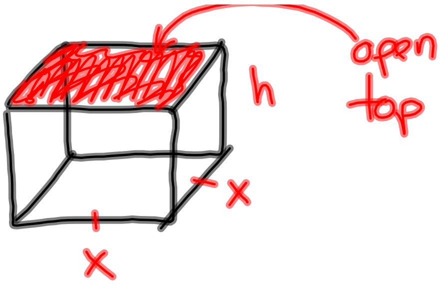
$S' = 3y^2 - 6y + 8y - 16$

$S' = 3y(y-2) + 8(y-2)$

$S' = (3y+8)(y-2)$

CV: $y = -\frac{8}{3}$, $\boxed{2}$





$$x^2h = 4000$$

$$h = \boxed{\frac{4000}{x^2}}$$

$$h = \frac{4000}{(20)^2}$$

$$h = 10 \text{ cm}$$

$\therefore$ The dimensions that minimize the surface area are $20 \times 20 \times 10$

$$A = x^2 + 4xh$$

$$A = x^2 + 4\cancel{x}\left[\frac{4000}{x^{\cancel{2}}}\right]$$

$$A = x^2 + 16000x^{-1}$$

$$A' = 2x - \frac{16000}{x^2}$$

$$A' = \frac{2x^3 - 16000}{x^2}$$

$$2x^3 - 16000 = 0$$

$$2x^3 = 16000$$

$$x^3 = 8000$$

$$x = 20$$

CV: $x = 0, \boxed{20}$

	$-$		$-$		$+$
(-1)		0	(1)	20	(21)

↑ min

5 Find the point on the parabola $2y = x^2$ that is closest to the point $(-4, 1)$

$y = \frac{x^2}{2}$ $\qquad$ x_1 $\quad$ y_1

$$d = \sqrt{(x-x_1)^2 + (y-y_1)^2}$$

$$d = \sqrt{(x+4)^2 + (y-1)^2}$$

$$d = \sqrt{(x+4)^2 + \left(\tfrac{1}{2}x^2 - 1\right)^2}$$

$$f(x) = (x+4)^2 + \left(\tfrac{1}{2}x^2 - 1\right)^2$$

$$f'(x) = 2(x+4)(1) + 2\left(\tfrac{1}{2}x^2 - 1\right)(x)$$

$$f'(x) = 2x + 8 + x^3 - 2x$$

$$f'(x) = x^3 + 8$$

$$x^3 = -8$$

$$\boxed{x = -2}$$

$$y = \frac{x^2}{2}$$

$$y = \frac{(-2)^2}{2}$$

$$y = 2$$

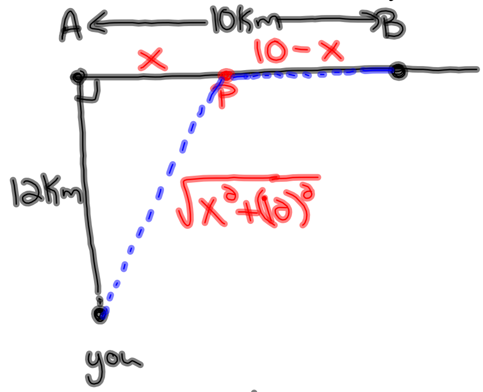
$(-2, 2)$

$-$ $\qquad$ $+$

(-3) $\quad -2 \quad$ (1)

↑ min

You are in a dune buggy in the desert 12km due south of the nearest point A on a straight east-west road. You wish to get to point B on the road 10km east of point A. If your dune buggy can average 15km/h travelling over the desert, and 39km/h travelling on the road, toward what point on the road should you head to in order to minimize your travel time to B?



Let x = distance from A to P

$$T = \frac{d}{s}$$

$$T = \frac{\sqrt{x^2+144}}{15} + \frac{10-x}{39}$$

$$T = \frac{1}{15}(x^2+144)^{1/2} + \frac{10}{39} - \frac{x}{39}$$

$$T' = \frac{1}{30}(x^2+144)^{-1/2}(2x) + 0 - \frac{1}{39}$$

$$T' = \frac{x(x^2+144)^{-1/2}}{15} - \frac{1}{39}$$

$$0 = \frac{x}{15\sqrt{x^2+144}} - \frac{1}{39}$$

$$\frac{1}{39} = \frac{x}{15\sqrt{x^2+144}}$$

square both

$$(39x)^2 = (15\sqrt{x^2+144})^2$$

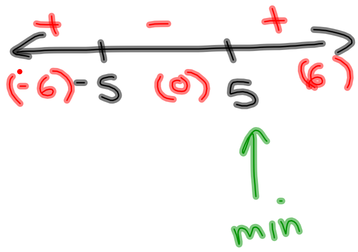
$$1521x^2 = 225(x^2+144)$$

$$1521x^2 = 225x^2 + 32400$$

$$1296x^2 = 32400$$

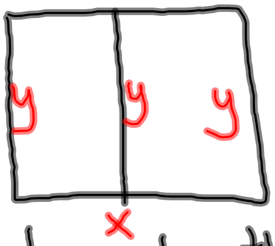
$$x^2 = 25$$

$$x = \pm 5$$



∴ Head to a point 5km east of A

You have 400 m of fencing to construct a rectangular pen that will be divided into 2 sections of equal size. Find the dimensions that would maximize the area of the whole pen.



Let x = length
Let y = width

$$P = 2x + 3y$$
$$400 = 2x + 3y$$
$$400 - 2x = 3y$$
$$\boxed{\frac{400-2x}{3} = y}$$

$$A = xy$$
$$A = x\left[\frac{400-2x}{3}\right]$$
$$A = \frac{400x - 2x^2}{3}$$
$$A = \frac{400}{3}x - \frac{2}{3}x^2$$
$$A' = \frac{400}{3} - \frac{4}{3}x$$
$$0 = \frac{400}{3} - \frac{4}{3}x$$
$$\frac{4x}{3} = \frac{400}{3}$$
$$12x = 1200$$
$$x = 100\text{m}$$

$$y = \frac{400 - 2(100)}{3}$$
$$y = \frac{200}{3}$$
$$y = 66.\overline{6}\ \text{m}$$

Find the points on the parabola $y = 6 - x^2$ that are closest to the point $(0, 3)$

$$d = \sqrt{(x - x_1)^2 + (y - y_1)^2}$$

$$d = \sqrt{(x - 0)^2 + (6 - x^2 - 3)^2}$$

$$d = \sqrt{x^2 + (3 - x^2)^2}$$

$$d = \sqrt{x^2 + 9 - 6x^2 + x^4}$$

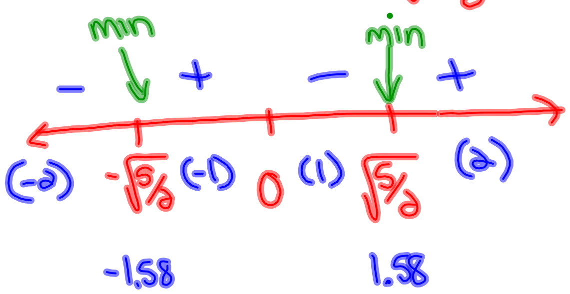
$$d = \sqrt{x^4 - 5x^2 + 9}$$

$$f(x) = x^4 - 5x^2 + 9$$

$$f'(x) = 4x^3 - 10x$$

$$f'(x) = 2x(2x^2 - 5)$$

$2x = 0$ | $2x^2 - 5 = 0$
$x = 0$ | $x^2 = 5/2$
| $x = \pm\sqrt{5/2}$



$$y = 6 - \left(-\sqrt{5/2}\right)^2$$

$$y = 6 - \frac{5}{2} = \frac{7}{2}$$

$$y = 6 - \left(\sqrt{5/2}\right)^2$$

$$y = 6 - \frac{5}{2} = \frac{7}{2}$$

The points are $\left(-\sqrt{5/2}, 7/2\right)$ and $\left(\sqrt{5/2}, 7/2\right)$

Homework