

$$\textcircled{1} \text{ b) } f(x) = 3x^5 - 6x^4 + 2$$

$$f'(x) = 15x^4 - 24x^3$$

$$\text{d) } g(x) = x^2 - \frac{2}{x^2} = x^2 - 2x^{-2}$$

$$g'(x) = 2x + 4x^{-3} = 2x + \frac{4}{x^3}$$

$$\text{j) } F(x) = \sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}$$

$$= x^{1/2} + x^{1/3} + x^{1/4}$$

$$F'(x) = \frac{1}{2}x^{-1/2} + \frac{1}{3}x^{-2/3} + \frac{1}{4}x^{-3/4}$$

$$= \frac{1}{2x^{1/2}} + \frac{1}{3x^{2/3}} + \frac{1}{4x^{3/4}}$$

$$= \frac{1}{2\sqrt{x}} + \frac{1}{3\sqrt[3]{x^2}} + \frac{1}{4\sqrt[4]{x^3}}$$

$$\text{k) } u(t) = a + \frac{b}{t} + \frac{c}{t^2} = a + bt^{-1} + ct^{-2}$$

$$u'(t) = -bt^{-2} - 2ct^{-3} = \frac{-b}{t^2} - \frac{2c}{t^3}$$

$$\text{l) } v(r) = \sqrt{r}(2+3r) = r^{1/2}(2+3r)$$

$$= 2r^{1/2} + 3r^{3/2}$$

$$v'(r) = r^{-1/2} + \frac{9}{2}r^{1/2} = \frac{1}{r^{1/2}} + \frac{9}{2}r^{1/2}$$

$$= \frac{1}{\sqrt{r}} + \frac{9\sqrt{r}}{2}$$

$$\textcircled{4} b) f(x) = \frac{2x-1}{4x} \quad f(x+h) = \frac{2(x+h)-1}{4(x+h)}$$

$$f(x+h) = \frac{2x+2h-1}{4x+4h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{2x+2h-1}{4x+4h} - \frac{2x-1}{4x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{4x(2x+2h-1) - (2x-1)(4x+4h)}{4xh(4x+4h)}$$

$$= \lim_{h \rightarrow 0} \frac{8x^2 + 8xh - 4x - (8x^2 + 8xh - 4x - 4h)}{4xh(4x+4h)}$$

$$= \lim_{h \rightarrow 0} \frac{4h}{4xh(4x+4h)} = \frac{1}{4x^2}$$

$$\textcircled{6} b) y = 2x^3 - 6\sqrt{x} \quad \text{at } (4, 20)$$

$$y = 2x^3 - 6x^{1/2}$$

$$\textcircled{1} y' = 4x - 3x^{-1/2}$$

$$y' = 4x - \frac{3}{\sqrt{x}}$$

$$\textcircled{2} y'(4) = 4(4) - \frac{3}{\sqrt{4}}$$

$$= 16 - \frac{3}{2}$$

$$= \frac{32}{2} - \frac{3}{2}$$

$$= \frac{29}{2} \leftarrow m$$

$$\textcircled{3} y - y_1 = m(x - x_1)$$

$$y - 20 = \frac{29}{2}(x - 4)$$

$$y - 20 = \frac{29x}{2} - 58$$

$$2y - 40 = 29x - 116$$

$$0 = 29x - 2y - 76$$

$\textcircled{7}$ Find the point on the curve $y = x\sqrt{x}$ where the tangent line is parallel to the line $6x - y = 4$

$$\textcircled{1} y = x\sqrt{x} = x(x^{1/2}) = x^{3/2}$$

$$y' = \frac{3}{2}x^{1/2}$$

$$\textcircled{2} 6x - y = 4$$

$$6x - 4 = y$$

$$\uparrow m = 6$$

$$\textcircled{3} \frac{3\sqrt{x}}{2} = 6$$

$$3\sqrt{x} = 12$$

$$\sqrt{x} = 4$$

$$x = 16$$

$$y = x\sqrt{x}$$

$$y = (16)\sqrt{16}$$

$$y = 64$$

$$\textcircled{4} \text{ d) } y = \sqrt[3]{x}, \quad (-8, -2)$$

x_1, y_1

① Find Derivative

$$y = x^{1/3}$$

$$y' = \frac{1}{3} x^{-2/3} = \frac{1}{3x^{2/3}}$$

② Fill in x value and solve for the slope: "m"

$$y' = \frac{1}{3(-8)^{2/3}} = \frac{1}{3(4)} = \frac{1}{12}$$

↑
slope of tangent

③ Use $y - y_1 = m(x - x_1)$

$$y + 2 = \frac{1}{12}(x + 8)$$

$$12 \cdot (y + 2) = \frac{12}{12} x + \frac{8}{12}$$

$$12y + 24 = x + 8$$

$$0 = x - 12y - 16$$

$$\textcircled{3} \text{ c) } y = x + \frac{6}{x}, \quad (\overset{x_1}{2}, \overset{y_1}{5}) \quad m = -\frac{1}{2}$$

① Find Derivative

$$y = x + 6x^{-1}$$

$$y' = 1 - 6x^{-2}$$

$$y' = 1 - \frac{6}{x^2}$$

③ Find the equation:

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -\frac{1}{2}(x - 2)$$

$$2y - 10 = -1(x - 2)$$

$$2y - 10 = -x + 2$$

② Sub in x-value ($x=2$)

$$y' = 1 - \frac{6}{(2)^2}$$

$$y' = \frac{4}{4} - \frac{6}{4}$$

$$y' = \frac{-2}{4} = -\frac{1}{2}$$

Slope of
the tangent
"m"

$$x + 2y - 12 = 0$$

$$\textcircled{4} \text{ a) } y = x^5 \quad x = 2 \quad \text{Point } (2, 32)$$

$$y = (2)^5$$

$$y = 32$$

$$m = 80$$

① Find Derivative

$$y' = 5x^4$$

② Sub in x-value ($x=2$)

$$y' = 5(2)^4$$

$$y' = 5(16)$$

$$y' = \boxed{80}$$

↑
Slope

③ Find the equation:

$$y - y_1 = m(x - x_1)$$

$$y - 32 = 80(x - 2)$$

$$y - 32 = 80x - 160$$

$$-80x + y + 128 = 0$$

$$\boxed{80x - y - 128 = 0}$$

$$\textcircled{2} \quad y = x\sqrt{x}$$

$$y = (x)(x^{1/2})$$

$$y = x^{3/2}$$

$$y' = \frac{3}{2}x^{1/2}$$

$$y' = \frac{3\sqrt{x}}{2}$$

parallel to: $6x - y = 4$

$$-y = -6x + 4$$

$$y = 6x - 4$$

$$m = 6$$

$$6 = \frac{3\sqrt{x}}{2}$$

$$\frac{12}{3} = \frac{3\sqrt{x}}{3}$$

$$(4)^2 = (\sqrt{x})^2$$

$$16 = x$$

\therefore The point is
(16, 64)

$$y = x\sqrt{x}$$

$$y = (16)\sqrt{16}$$

$$y = 16(4)$$

$$y = 64$$

$$\textcircled{1} \text{ g) } y = \frac{x+1}{\sqrt{x}} = \frac{x+1}{x^{1/2}} = x^{-1/2}(x+1) \\ = x^{1/2} + x^{-1/2}$$

$$y' = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2}$$

$$y' = \frac{1}{2x^{1/2}} - \frac{1}{2x^{3/2}}$$

$$\textcircled{1} \text{ a) } f(x) = x^2 + 4x$$

$$f'(x) = 2x + 4$$

$$\text{b) } g(x) = x^2 - \frac{2}{x^2} = x^2 - 2x^{-2}$$

$$g'(x) = 2x + 4x^{-3}$$

$$g'(x) = 2x + \frac{4}{x^3}$$

$$\text{g) } y = \frac{x+1}{\sqrt{x}} = \frac{x+1}{x^{1/2}} = x^{-1/2}(x+1)$$

$$= x^{1/2} + x^{-1/2}$$

$$y' = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2}$$

$$\text{or } y' = \frac{1}{2x^{1/2}} - \frac{1}{2x^{3/2}}$$

$$\text{or } y' = \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x^3}}$$

$$\text{e) } h(x) = \sqrt{x} - 5x^4$$

$$= x^{1/2} - 5x^4$$

$$h'(x) = \frac{1}{2}x^{-1/2} - 20x^3$$

$$= \frac{1}{2x^{1/2}} - 20x^3$$

$$\textcircled{4} \text{ b) } y = 2\sqrt{x} \quad ; \quad (\underline{9}, \underline{6}) \quad ; \quad m = \frac{1}{3}$$

① Find Derivative

$$y = 2x^{1/2}$$

$$y' = 1x^{-1/2}$$

$$y' = \frac{1}{x^{1/2}} = \frac{1}{\sqrt{x}}$$

② Sub in x-value ($x=9$)

$$y' = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

↑
Slope of
the tangent
"m"

③ Find the Equation

$$y - y_1 = m(x - x_1)$$

$$3. \quad y - 6 = \frac{1}{3}(x - 9)$$

$$3y - 18 = 1(x - 9)$$

$$3y - 18 = x - 9$$

$$-x + 3y - 9 = 0$$

$$\boxed{x - 3y + 9 = 0}$$

$$\textcircled{7} \quad y = 3x^2$$

$$y' = 6x$$

$$24 = 6x$$

$$\boxed{4 = x}$$

$$y = 3(4)^2$$

$$y = 3(16)$$

$$\boxed{y = 48}$$

\therefore The point is $(4, 48)$

$$\textcircled{4} \quad \boxed{f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}}$$

$$b) \quad f(x) = \frac{2x-1}{4x} \quad \Bigg| \quad f(x+h) = \frac{2(x+h)-1}{4(x+h)} \\ = \frac{2x+2h-1}{4x+4h}$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{2x+2h-1}{4x+4h} - \frac{2x-1}{4x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{4x(2x+2h-1) - (2x-1)(4x+4h)}{h(4x)(4x+4h)} \\ &= \lim_{h \rightarrow 0} \frac{8x^2 + 8xh - 4x - (8x^2 + 8xh - 4x - 4h)}{h(4x)(4x+4h)} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{8x^2} + \cancel{8xh} - \cancel{4x} - \cancel{8x^2} - \cancel{8xh} + \cancel{4x} + 4h}{h(4x)(4x+4h)} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{4h}}{\cancel{h}(4x)(4x+4h)} = \frac{4}{(4x)^2} = \frac{4}{16x^2} = \boxed{\frac{1}{4x^2}} \end{aligned}$$