Warm Up

For the following rational function find the location of the oblique *asymptote.*

$$f(x)=\frac{2x^3-x^2+3x-4}{x^2+2x+3}$$

$$2x-5$$

$$x^2+2x+3\,\big)\,2x^3-x^2+3x-4$$

$$-(2x^3+4x^2+6x)$$

$$-5x^2-3x-4$$

$$-(-5x^2-10x-15)$$

$$7x+11 \text{ R}$$

OA: $y = 2x - 5$

$m = \frac{2}{1}$ $b = -5$

Questions from Homework

Graphing Rational Functions

$$f(x)=\frac{x^2-9}{x^2} = \frac{(x-3)(x+3)}{x^2}$$

① Roots:

$(x-3)(x+3)=0$

$x-3=0 \mid x+3=0$

$x=3 \mid x=-3$

(3,0) (-3,0)

② y int

$y=\frac{0-9}{0}$

$y=-\frac{9}{0}$

No y-int.

③ VA:

$x^2=0$

$x=0$

④ HA:

$\lim_{x\to\infty} \frac{x^2-9}{x^2} = 1$

$y=1$

⑤ No Holes

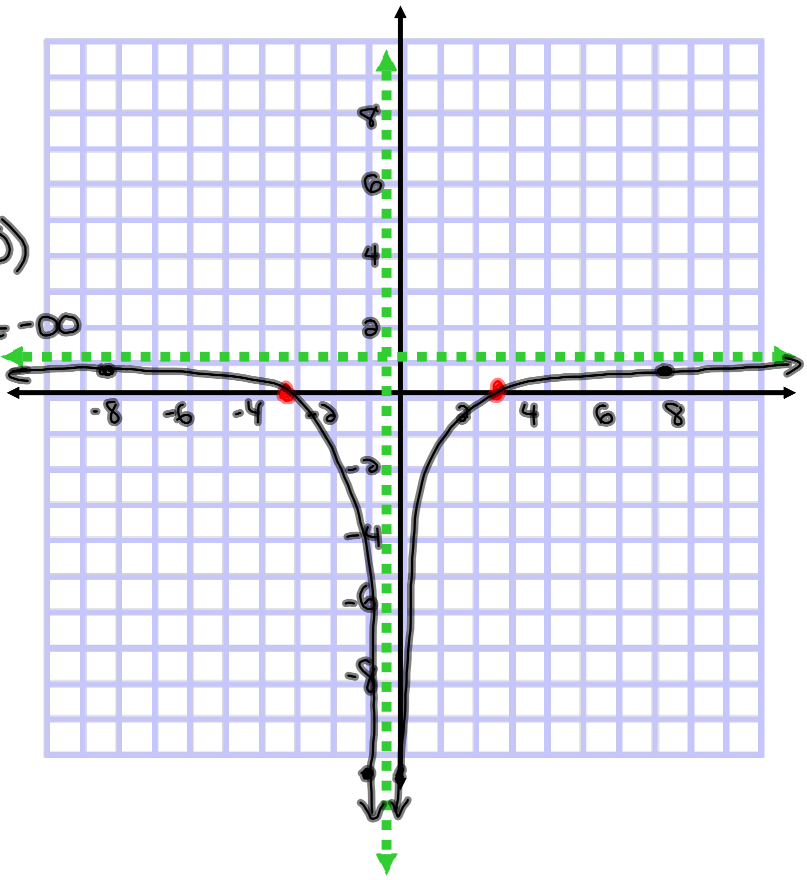
⑥ Check behaviour near the VA: (x=0)

$\lim_{x\to 0^-} \frac{(x-3)(x+3)}{x^2} = \frac{(-)(+)}{(+)} = -\infty$

(-0.01)

$\lim_{x\to 0^+} \frac{(-)(+)}{(+)} = -\infty$

(0.01)



Graphing Rational Functions

$$f(x)=\frac{x^2+2x-3}{x+1} = \frac{(x+3)(x-1)}{x+1}$$

① Roots:

$x=-3, 1$

$(-3,0)$ & $(1,0)$

② y int:

$y=\frac{-3}{1}=-3$

$(0,-3)$

③ VA:

$x=-1$

④ Holes:

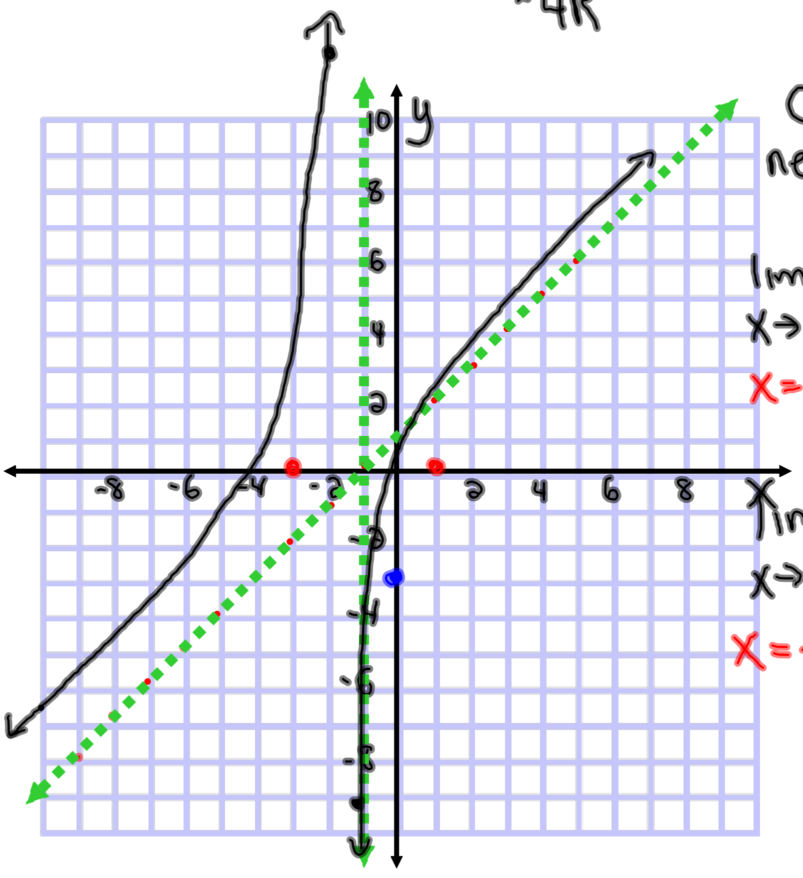
None

⑤ OA:

$$\begin{array}{r} x+1 \\ x+1 \,\big)\overline{\,x^2+2x-3} \\ -(x^2+x) \\ \hline x-3 \\ -(x+1) \\ \hline -4R \end{array}$$

OA:

$y=x+1$ $\quad m=\frac{1}{1}$ $\quad b=1$



Check Behaviour near the Vertical Asym

$$\lim_{x\to -1^-} \frac{(+)(-)}{(-)} = +\infty$$

$x=-1.01$

$$\lim_{x\to -1^+} \frac{(+)(-)}{(+)} = -\infty$$

$x=-0.99$

$$y = \frac{x^2+5x+6}{(x-1)} = \frac{(x+2)(x+3)}{(x-1)}$$

① Roots: $x = -2, -3$

② y int: $y = \frac{6}{-1} = -6$

③ VA: $x = 1$

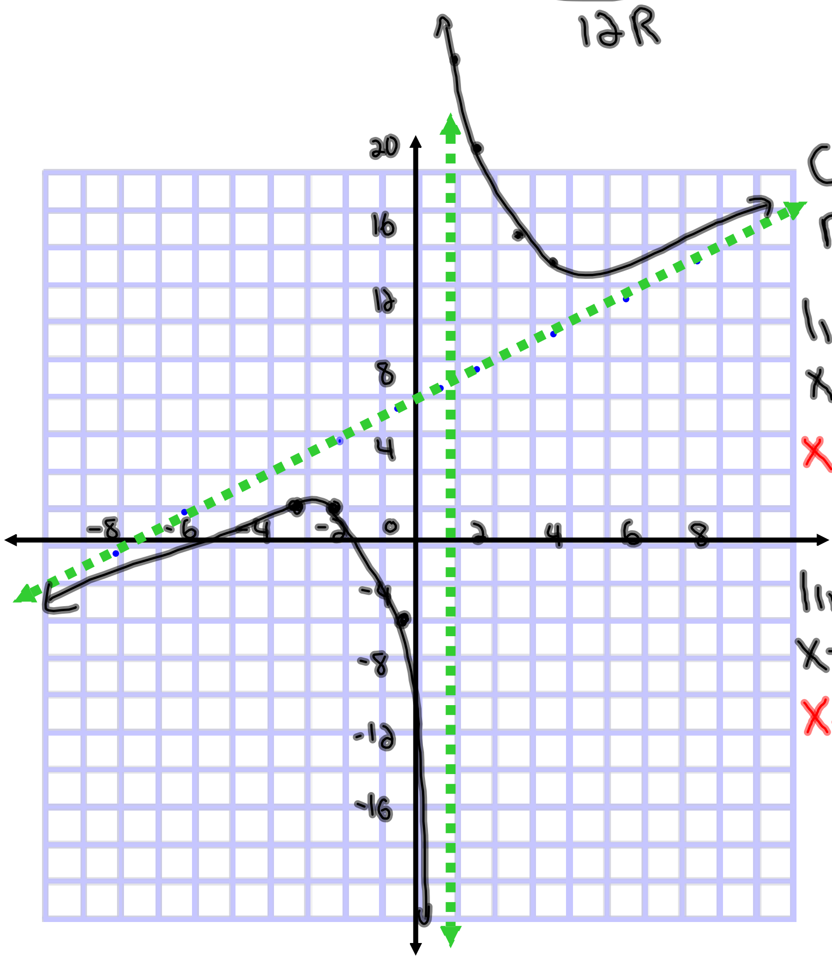
④ Holes: None

⑤ OA:

$$\begin{array}{r} x+6 \\ x-1\,\overline{)\,x^2+5x+6} \\ -(x^2-x)\quad\downarrow \\ \hline 6x+6 \\ -(6x-6) \\ \hline 12R \end{array}$$

OA: $y = x+6$

$m = \frac{1}{1}$ $b = 6$



Check behaviour near VA:

$$\lim_{x \to 1^-} \frac{(+)(+)}{(-)} = -\infty$$

$x = 0.99$

$$\lim_{x \to 1^+} \frac{(+)(+)}{(+)} = +\infty$$

$x = 1.01$

Rational Functions Continued

We will explore the properties of rational functions of this form so we can predict the locations of vertical, horizontal, and oblique asymptotes. We will also be able to identify the roots of the function and any other points of discontinuity (holes).

$$y = \frac{(x+2)(x+3)}{(x-1)}$$

$$y = \frac{(x+2)(x+3)}{(x+2)}$$

$$y = \frac{x^2+2x-3}{x^2+3x-4}$$

$$f(x) = \frac{4x}{x^2-4}$$

Roots
Are given by the zeroes of the numerator.

Vertical Asymptotes:
Are given by the zeroes of the denominator

Horizontal Asymptotes:
If the numerator and denominator have the same degree, then the horizontal asymptote is given by the quotient of the leading coefficients of the numerator and denominator.

If the degree of the denominator is greater than that of the numerator, then the horizontal asymptote is given by $y = 0$.

If the degree of the denominator is less than that of the numerator, **then there is no horizontal asymptote (*an oblique asymptote exists).***

Holes
Occur when the same factor is in the numerator and the denominator.

Oblique Asymptotes:
A line with a finite, non-zero slope that a graph approaches at extreme values but never crosses. They occur when the degree of the numerator is one greater than the degree of the denominator and can be determined by dividing the numerator by the denominator (ignoring the remainder).

We can use the factor theorem (long division) or synthetic substitution

This table shows whether a factor of a rational function results in a vertical asymptote, a root, or another point of discontinuity (hole).

Type of Factor	Vertical Asymptote	Hole	Zero of Function
Appears in numerator only			Zero of factor
Appears in denominator only	Zero of factor		
Appears in numerator and (to equal or lesser power) denominator		Zero of common factor	
Appears in numerator and (to a greater power) denominator	Zero of common factor		

This table below shows whether a rational function has a horizontal or oblique asymptote.

Type of Equation	Horizontal Asymptote	Oblique Asymptote
Degree of numerator is equal to degree of denominator	Given by quotient of leading coefficients in numerator and denominator	
Degree of numerator is less than degree of denominator	$y = 0$	
Degree of numerator is one more than degree of denominator		The equation can be found by examining the quotient of numerator and denominator (ignoring the remainder)

Homework

Sketch the following rational functions:

$$f(x) = \frac{x^2 + 5x + 6}{(x+2)^2}$$

$$f(x) = \frac{x^2 - 4}{x^2 - 1}$$

$$y = \frac{x^2 + 2x - 3}{x^2 + 3x - 4}$$