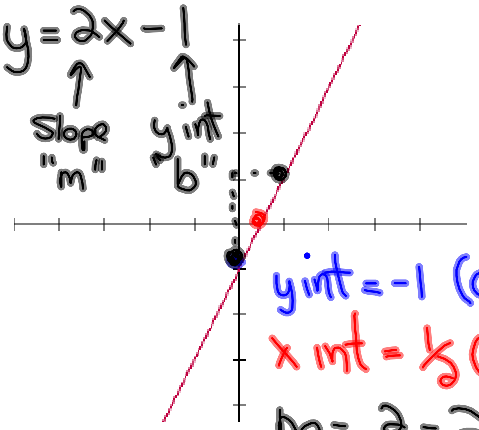
Catalog of Essential Functions

1. Linear

$y = 2x - 1$

Slope "m"   y int "b"



y int $= -1$ $(0,-1)$

x int $= \frac{1}{2}$ $(\frac{1}{2}, 0)$

$m = \frac{2}{1} = 2$

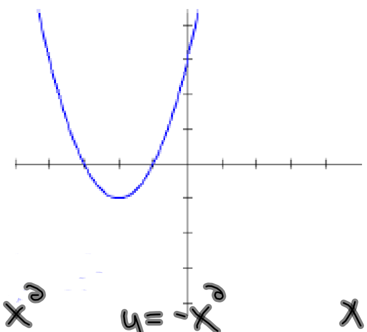
Straight Line

Equation will be degree one

Should be able to identify the ***slope, intercepts, and equation*** from the graph

$y = x$

2. Quadratic



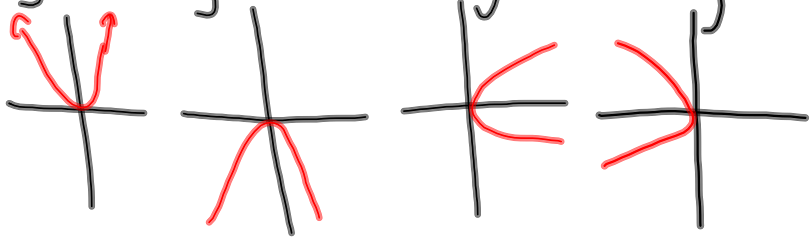
Parabola (U-Shaped)

Either *y or x* will be squared (not both!)

* Should know the 4 basic quadratic functions

Should be able to apply transformations to the basic quadratic functions

* $y = x^2$ $y = -x^2$ $x = y^2$ $x = -y^2$

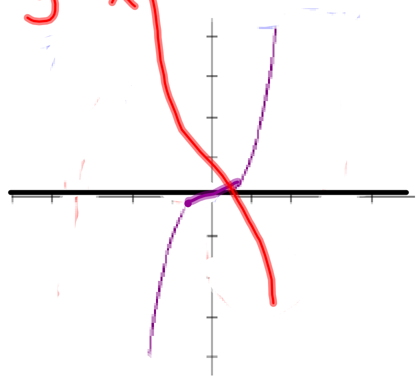


$y = x^2 \rightarrow$

x	y
-2	4
-1	1
0	0
1	1
2	4

3. Cubic

$y = -x^3$



S-Shaped

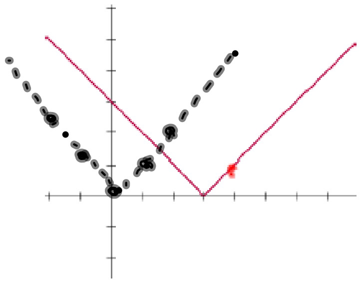
We will work with functions that are raised to the third power

$y = x^3$

x	y
-2	-8
-1	-1
0	0
1	1
2	8

Catalog of Essential Functions

4. Absolute Value



V-Shaped

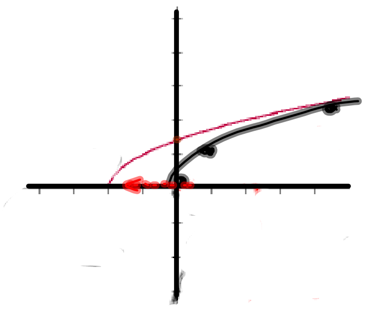
Equation will have a variable within the absolute value bars

Should be able to apply transformations to the basic absolute value function

$y=|x|$

x	y
-2	2
-1	1
0	0
1	1
2	2

5. Square Root



Half Parabola

Equation will have a variable under the square root sign

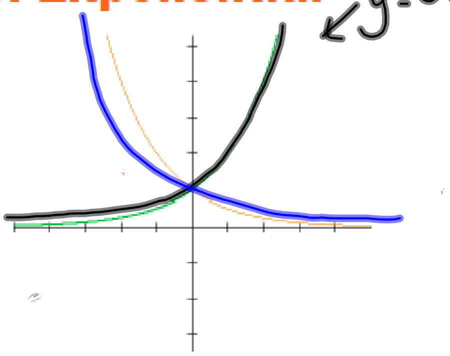
Should be able to apply transformations to the basic square root function

$y=\sqrt{x}$

x	y
0	0
1	1
4	2
9	3

6. Exponential

$y=2x$



Steadily increasing or decreasing

Base will be a number and variable will appear in the exponent ex: $y=2^x$

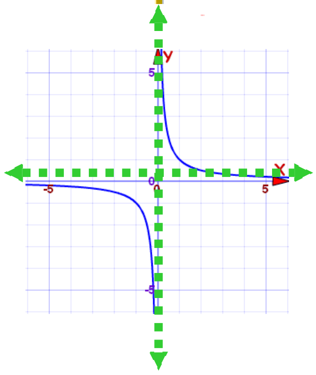
Should be able to identify the ***horizontal asymptote***

$y=2^x$

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4

Catalog of Essential Functions

7. Reciprocal



Will have two branches

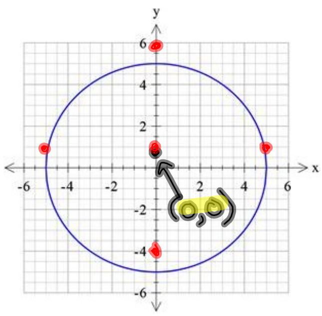
Equation will have a variable within the denominator of a rational expression

Should be able to identify the ***vertical and horizontal asymptotes***

$y = \frac{1}{x}$

x	y
-2	-1/2
-1	-1
0	undefined
1	1
2	1/2

8. Circle



- General form: $(x-h)^2 + (y-k)^2 = r^2$

  * center: (h, k) (0,0)

  * radius $= r$ 5

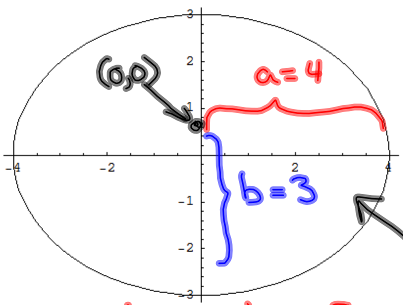
- Be able to identify the function that would describe either just the top or bottom of the circle.

$x^2 + y^2 = r^2$

$x^2 + y^2 = 5^2$

$x^2 + y^2 = 25$

9. Ellipse



major axis = 8

minor axis = 6

- General form: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Where... → (0,0)

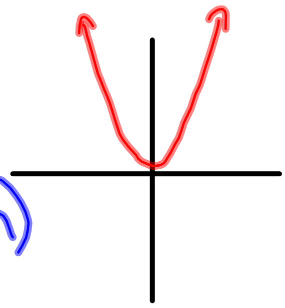
- Center: (h, k)
- a > b
- If a is the denominator of the "y" term the ellipse will have a vertical major axis.

$\frac{(x-0)^2}{4^2} + \frac{(y-0)^2}{3^2} = 1$

$$\boxed{\frac{x^2}{16} + \frac{y^2}{9} = 1}$$

Transformations:

New Functions From Old Functions



Translations
(slide the graph)

Stretches

Reflections

Translations

horizontal vertical h units right

Focus on...

k units up

- determining the effects of h and k in $y - k = f(x - h)$ on the graph of $y = f(x)$

$y = f(x-h) + k$

- sketching the graph of $y - k = f(x - h)$ for given values of h and k, given the graph of $y = f(x)$
- writing the equation of a function whose graph is a vertical and/or horizontal translation of the graph of $y = f(x)$

Translation

- To *translate* or *shift* a graph is to move it up, down, left, or right without changing its shape.
- Translation is summarized by the following table and illustration:

Vertical and Horizontal Shifts Suppose $c > 0$. To obtain the graph of

$y = f(x) + k$, shift the graph of $y = f(x)$ a distance k units upward
$y = f(x) - k$, shift the graph of $y = f(x)$ a distance k units downward
$y = f(x - h)$, shift the graph of $y = f(x)$ a distance h units to the right
$y = f(x + h)$, shift the graph of $y = f(x)$ a distance h units to the left

Ex: $y = x^2$

$y = (x+\underline{5})^2$

$h = -5 \rightarrow$ shift 5 units left

$k = 0$

Ex: $y = |x-\underline{3}| + \underline{2}$

$h = 3 \rightarrow$ Shift 3 units right

$k = 2 \rightarrow$ Shift 2 units up

Translations illustrated...

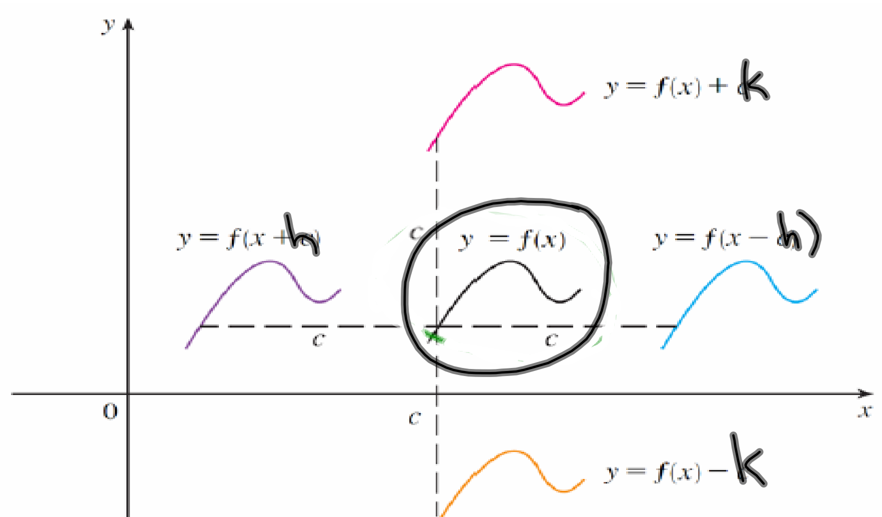


FIGURE 1
Translating the graph of f

Using Mapping Notation to Describe Transformations:

*Think of this as a set of instructions to follow to transform a graph.

$y=x^2$

x	$y=x^2$
−3	9
−2	4
−1	1
0	0
1	1
2	4
3	9

(x, y)

Vertical

$y=x^2+2$

x	$y=x^2+2$
−3	11
−2	6
−1	3
0	2
1	3
2	6
3	11

$(x, y) \to (x, y+2)$

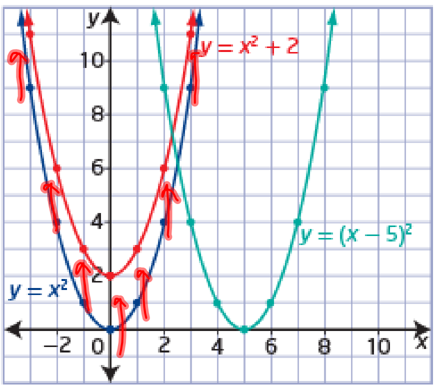
Horizontal

$y=(x-5)^2$

x	$y=(x-5)^2$
2	9
3	4
4	1
5	0
6	1
7	4
8	9

$(x, y) \to (x+5, y)$

Graph Translations of the Form $y-k=f(x)$ and $y=f(x-h)$



Ex: $y=(x+4)^2-6$ ← $y=x^2$

h=−4 → Left

k=−6 → Down

$(x, y) \to (x-4, y-6)$

$y+10=\frac{1}{x-7}$ ← $y=$

or $y=\frac{1}{x-7}-10$

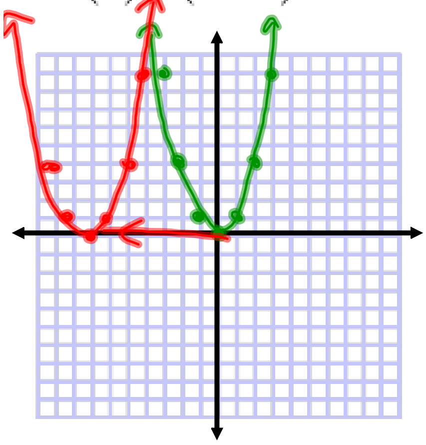
h=7 → Right

k=−10 → Down

Identify the translations for each of the following...

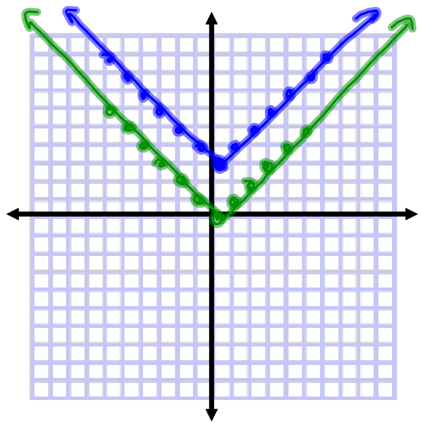
$h=-7$

$f(x)=(x+7)^2$ Base: $y=x^2$

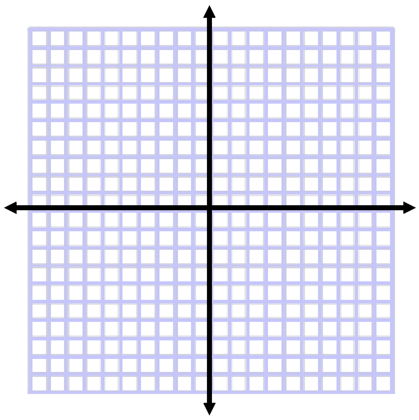


$K=3$

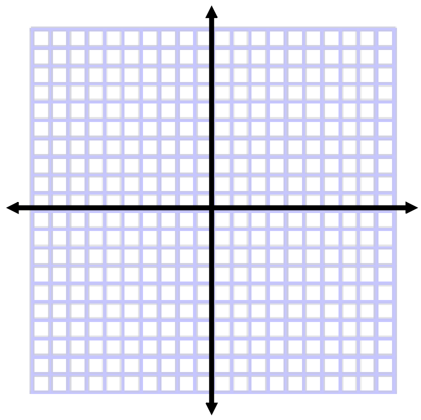
$f(x)=|x|+3$ Base: $y=|x|$



$f(x)=\sqrt{x-3}-2$

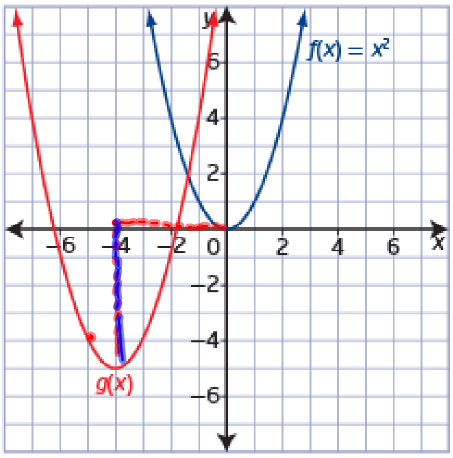


$f(x)=\dfrac{1}{x-5}+7$



Determine the Equation of a Translated Function:

a)



$h = -4$

Left 4

$k = -5$

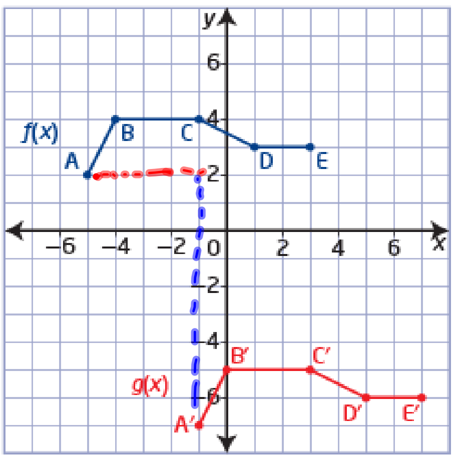
Down 5

$y = (x+4)^2 - 5$

or

$y + 5 = (x+4)^2$

b)



$h = 4$

Right 4

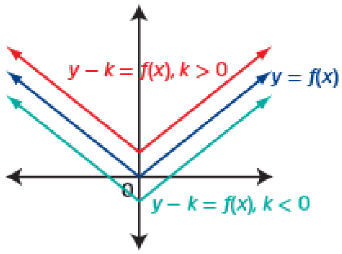
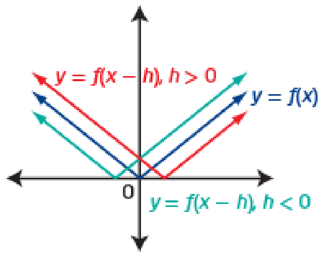
$k = -8$

Down 8

Function Notation

$g(x) = f(x-4) - 8$

- Translations are transformations that shift all points on the graph of a function up, down, left, and right without changing the shape or orientation of the graph.
- The table summarizes translations of the function $y = f(x)$.

Function	Transformation from $y = f(x)$	Mapping	Example
$y - k = f(x)$ or $y = f(x) + k$	A vertical translation If $k > 0$, the translation is up. If $k < 0$, the translation is down.	$(x, y) \rightarrow (x, y + k)$	
$y = f(x - h)$	A horizontal translation If $h > 0$, the translation is to the right. If $h < 0$, the translation is to the left.	$(x, y) \rightarrow (x + h, y)$	

- A sketch of the graph of $y - k = f(x - h)$, or $y = f(x - h) + k$, can be created by translating key points on the graph of the base function $y = f(x)$.

Homework

1-6 from page 4 of workbook.