Warm-Up

8. Copy and complete the table.

Translation	Transformed Function	Transformation of Points
vertical	$y = f(x) + 5$	$(x, y) \rightarrow (x, y + 5)$
H	$y = f(x + 7)$	$(x, y) \rightarrow (x - 7, y)$
H	$y = f(x - 3)$	$(x,y) \rightarrow (x+3, y)$
V	$y = f(x) - 6$	$(x,y) \rightarrow (x, y-6)$
horizontal and vertical	$y + 9 = f(x + 4)$	$(x,y) \rightarrow (x-4, y-9)$
horizontal and vertical	$y+6 = f(x-4)$	$(x, y) \rightarrow (x + 4, y - 6)$
H + V	$y-3 = f(x+2)$	$(x, y) \rightarrow (x - 2, y + 3)$
horizontal and vertical	$y = f(x - h) + k$	$(x,y) \rightarrow (x+h, y+k)$

Questions from Homework

① c) $y = f(x-17)+13$ or $y-13 = f(x-17)$

$h = 17$ $k = 13$

② Given $h = 2$ and $k = -5$ $\rightarrow$ $y-k = f(x-h)$

a) $y = x^2$ $y+5 = (x-2)^2$

b) $y = |x|$ $y+5 = |x-2|$

c) $y = \frac{1}{x}$ $y+5 = \frac{1}{x-2}$

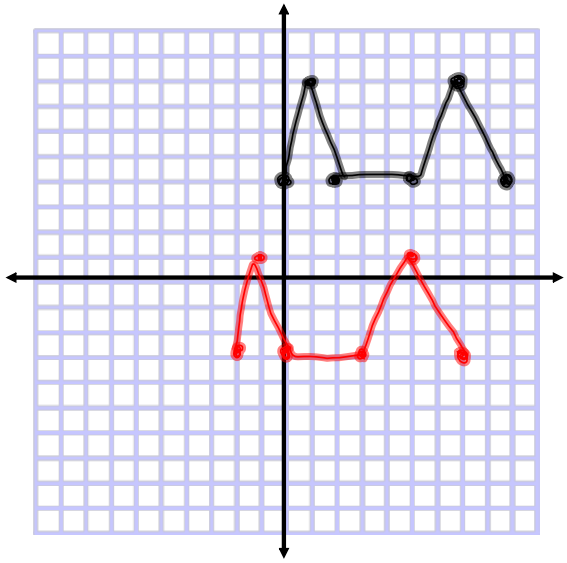
③ c) $y-10 = f(x+20)$ $h = -20$ $k = 10$

$(x,y) \rightarrow (x-20, y+10)$

④ a) $y+7 = f(x+2)$

$h = -2$

$k = -7$



$(x,y) \rightarrow (x-2, y-7)$

(0,4)	(-2,-3)
(1,8)	(-1,1)
(2,4)	(0,-3)
(5,4)	(3,-3)
(7,8)	(5,1)
(9,4)	(7,-3)

Transformations:

New Functions From Old Functions

Translations

Stretches

✓Reflections

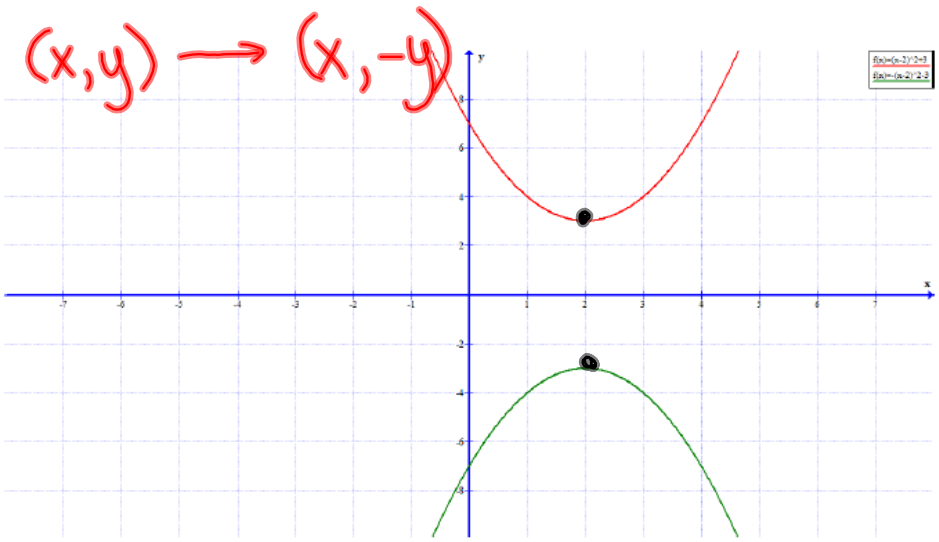
Reflections and Stretches

Focus on...

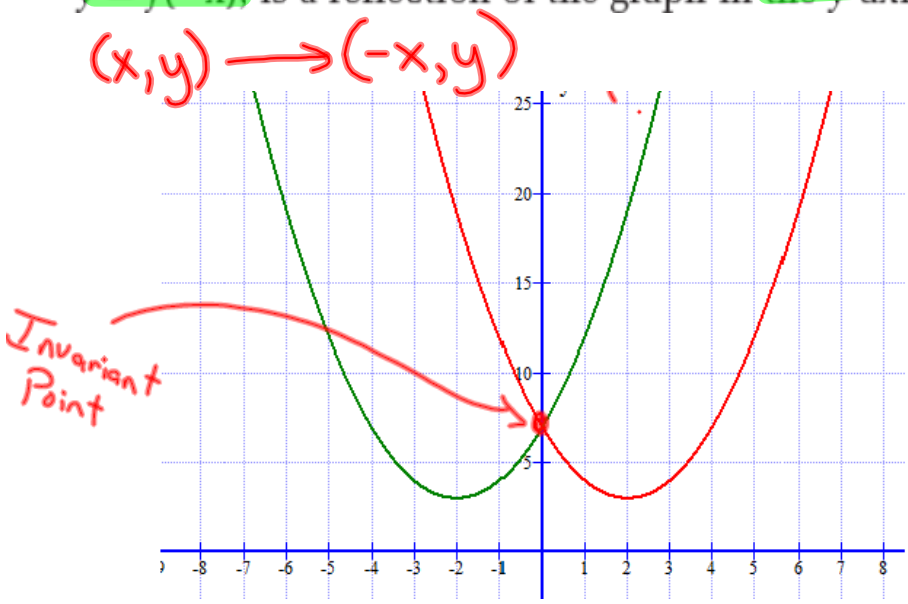
- developing an understanding of the effects of reflections on the graphs of functions and their related equations
- developing an understanding of the effects of vertical and horizontal stretches on the graphs of functions and their related equations

A **reflection** of a graph creates a mirror image in a line called the line of reflection. Reflections, like translations, do not change the shape of the graph. However, unlike translations, reflections may change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.



- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.



invariant point

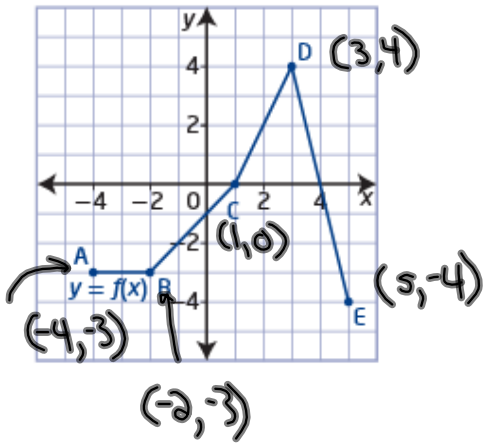
- a point on a graph that remains unchanged after a transformation is applied to it
- any point on a curve that lies on the line of reflection is an invariant point

Example 1

Compare the Graphs of $y = f(x)$, $y = -f(x)$, and $y = f(-x)$

a) Given the graph of $y = f(x)$, graph the functions $y = -f(x)$ and $y = f(-x)$.

b) How are the graphs of $y = -f(x)$ and $y = f(-x)$ related to the graph of $y = f(x)$?



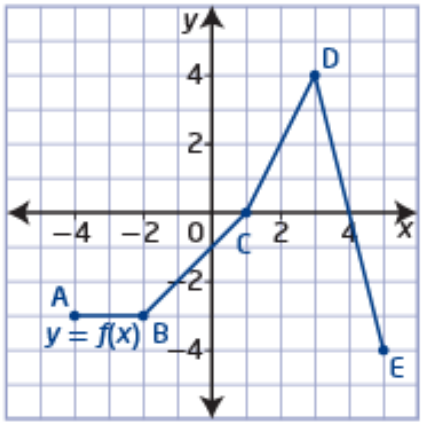
a) $y = -f(x)$

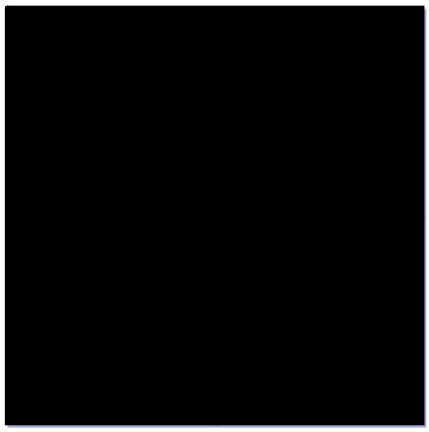
b) $y = f(-x)$

Remember...

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.

- Sketch $y = -f(x)$ on the axis below

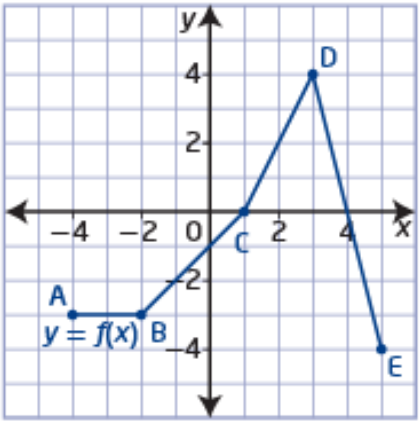




Remember...

- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

- Sketch $y = f(-x)$ on the axis below



stretch

- a transformation in which the distance of each *x*-coordinate or *y*-coordinate from the line of reflection is multiplied by some scale factor

- scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection

Vertical and Horizontal Stretches

A **stretch**, unlike a translation or a reflection, changes the shape of the graph. However, like translations, stretches do not change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

Vertical Stretch or Compression...

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.

Example 2

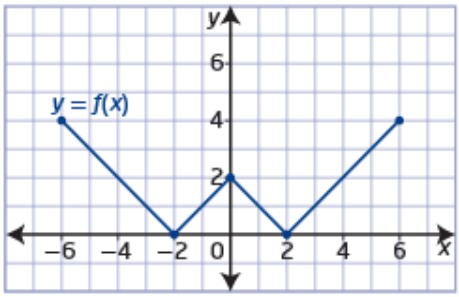
Graph $y = af(x)$

Given the graph of $y = f(x)$,

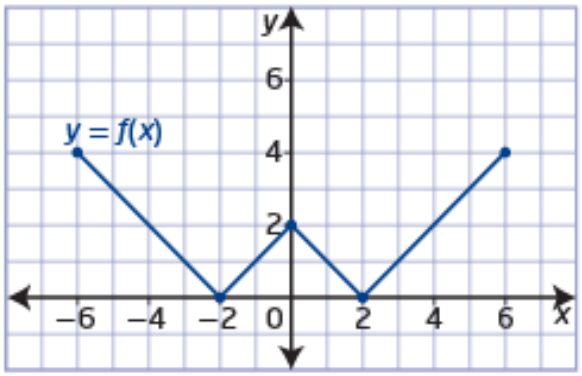
- transform the graph of $f(x)$ to sketch the graph of $g(x)$
- describe the transformation
- state any invariant points
- state the domain and range of the functions

a) $g(x) = 2f(x)$

b) $g(x) = \frac{1}{2}f(x)$



a) $g(x) = 2f(x)$



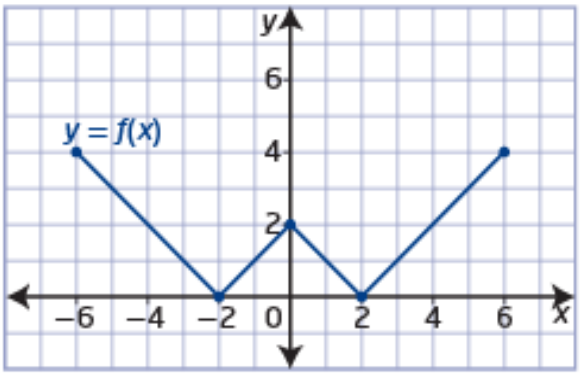
The invariant points are and

For $f(x)$, the domain is

and the range is

For $g(x)$, the domain is
and the range is

b) $g(x) = \frac{1}{2}f(x)$



The invariant points are and

For $f(x)$, the domain is

and the range is

For $g(x)$, the domain is
and the range is

Horizontal Stretch or Compression...

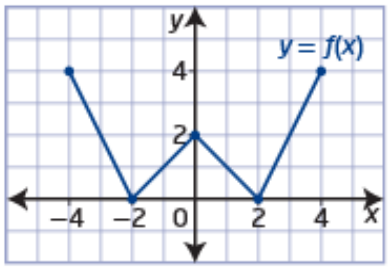
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

Example 3

Graph $y = f(bx)$

Given the graph of $y = f(x)$,

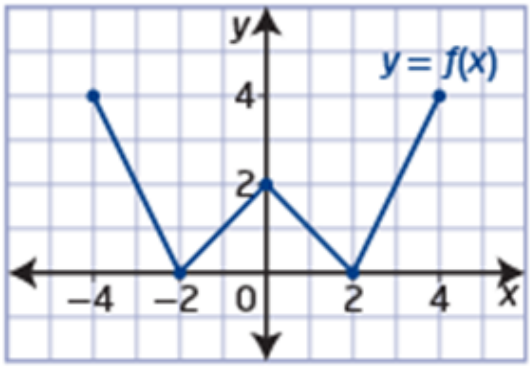
- transform the graph of $f(x)$ to sketch the graph of $g(x)$
- describe the transformation
- state any invariant points
- state the domain and range of the functions



a) $g(x) = f(2x)$

b) $g(x) = f\left(\frac{1}{2}x\right)$

a) $g(x) = f(2x)$

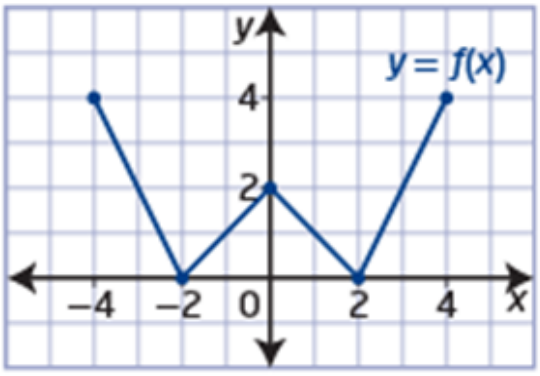


The invariant point is

For $f(x)$, the domain is
or and the range is
or

For $g(x)$, the domain is
or nd the range is
or

b) $g(x) = f\left(\frac{1}{2}x\right)$



The invariant point is

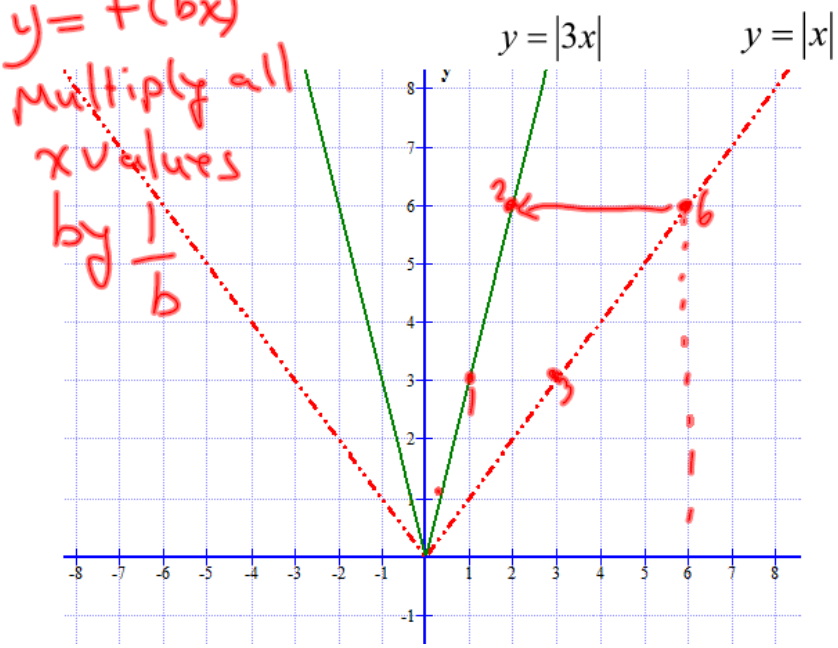
For $f(x)$, the domain is
and the range is

For $g(x)$, the domain is
and the range is

Horizontal Stretch or Compression...

$y = f(bx)$

Multiply all x values by $\frac{1}{b}$



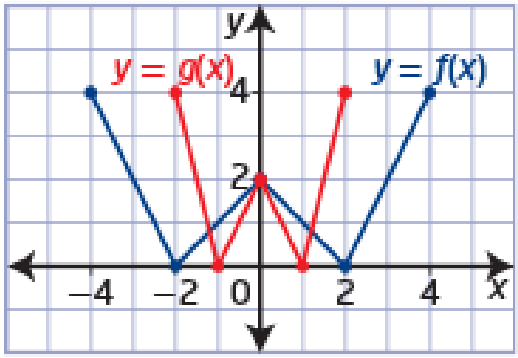
Horizontal Stretch or Compression...

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

$y = -3f(-2x) + 7$

Homework

Determine the Equation of a Translated Function:


y
y = g(x)
y = f(x)
4
2
−4
−2
0
2
4
x