Warm-Up

8. Copy and complete the table.

Translation	Transformed Function	Transformation of Points
vertical	$y = f(x) + 5$	$(x, y) \rightarrow (x, y + 5)$
H	$y = f(x + 7)$	$(x, y) \rightarrow (x - 7, y)$
H	$y = f(x - 3)$	$(x,y) \rightarrow (x+3, y)$
V	$y = f(x) - 6$	$(x,y) \rightarrow (x, y-6)$
horizontal and vertical	$y + 9 = f(x + 4)$	$(x,y) \rightarrow (x-4, y-9)$
horizontal and vertical	$y+6=f(x-4)$	$(x, y) \rightarrow (x + 4, y - 6)$
H + V	$y-3=f(x+2)$	$(x, y) \rightarrow (x - 2, y + 3)$
horizontal and vertical	$y = f(x - h) + k$	$(x,y) \rightarrow (x+h, y+k)$

Questions from Homework

① c) $y = f(x-17)+13$ or $y-13 = f(x-17)$

$h=17$ $k=13$

② Given $h=2$ and $k=-5$ $\rightarrow$ $y-k = f(x-h)$

a) $y = x^2$ b) $y = |x|$ c) $y = \frac{1}{x}$

$y+5 = (x-2)^2$ $y+5 = |x-2|$ $y+5 = \frac{1}{x-2}$

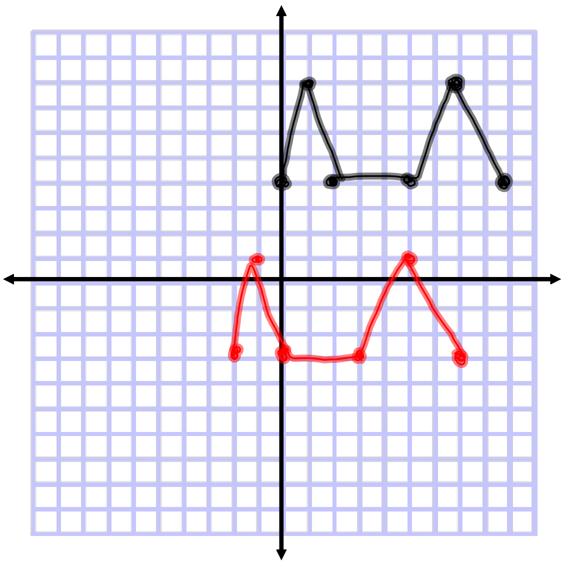
③ c) $y-10 = f(x+20)$ $h=-20$ $k=10$

$(x,y) \longrightarrow (x-20, y+10)$

④ a) $y+7 = f(x+2)$

$h=-2$

$k=-7$



$(x,y) \longrightarrow (x-2, y-7)$

(0,4)	(-2,-3)
(1,8)	(-1,1)
(2,4)	(0,-3)
(5,4)	(3,-3)
(7,8)	(5,1)
(9,4)	(7,-3)

Transformations:

New Functions From Old Functions

Translations

✓Stretches

✓Reflections

Reflections and Stretches

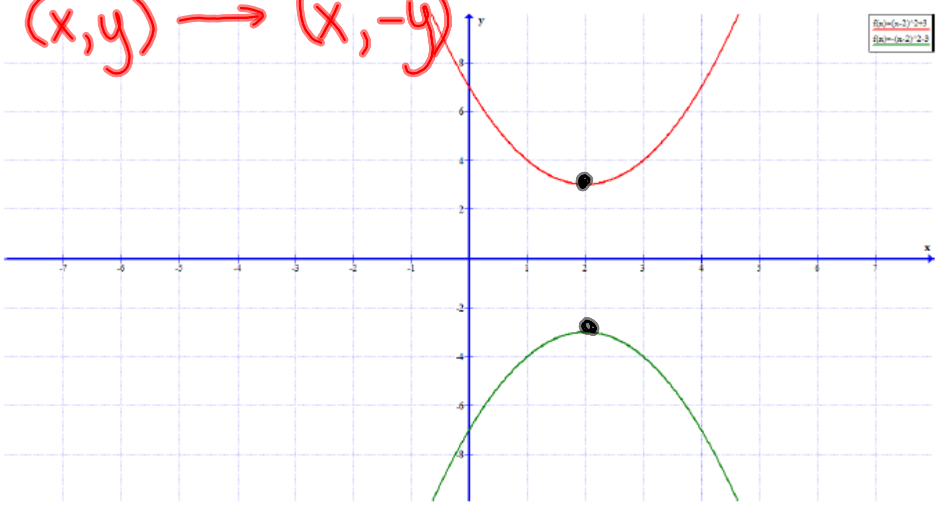
Focus on...

- developing an understanding of the effects of reflections on the graphs of functions and their related equations
- developing an understanding of the effects of vertical and horizontal stretches on the graphs of functions and their related equations

A **reflection** of a graph creates a mirror image in a line called the line of reflection. Reflections, like translations, do not change the shape of the graph. However, unlike translations, reflections may change the orientation of the graph.

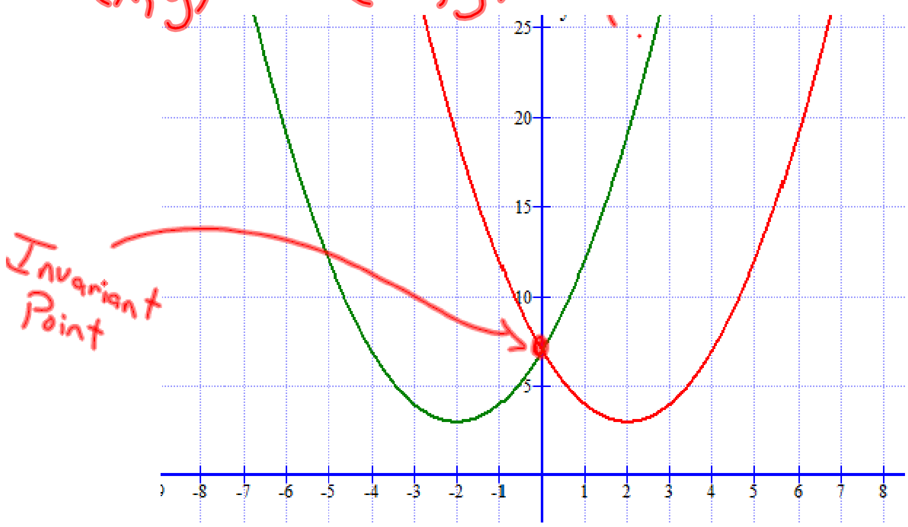
- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.

$(x, y) \longrightarrow (x, -y)$



- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

$(x, y) \longrightarrow (-x, y)$



invariant point

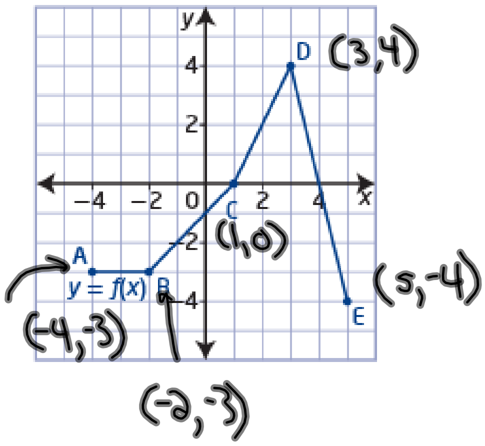
- a point on a graph that remains unchanged after a transformation is applied to it
- any point on a curve that lies on the line of reflection is an invariant point

Example 1

Compare the Graphs of $y = f(x)$, $y = -f(x)$, and $y = f(-x)$

a) Given the graph of $y = f(x)$, graph the functions $y = -f(x)$ and $y = f(-x)$.

b) How are the graphs of $y = -f(x)$ and $y = f(-x)$ related to the graph of $y = f(x)$?



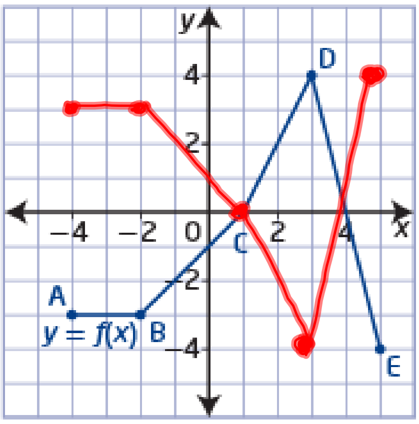
a) $y = -f(x)$

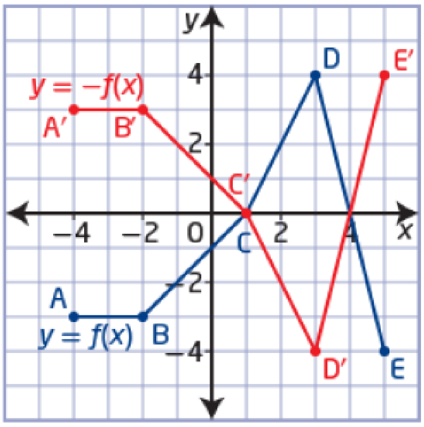
b) $y = f(-x)$

Remember...

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.

- Sketch $y = -f(x)$ on the axis below





$(x, y) \longrightarrow (x, -y)$

A $(-4,-3)$ $(-4,3)$

B $(-2,-3)$ $(-2,3)$

C $(1,0)$ $(1,0)$ ← Invariant point

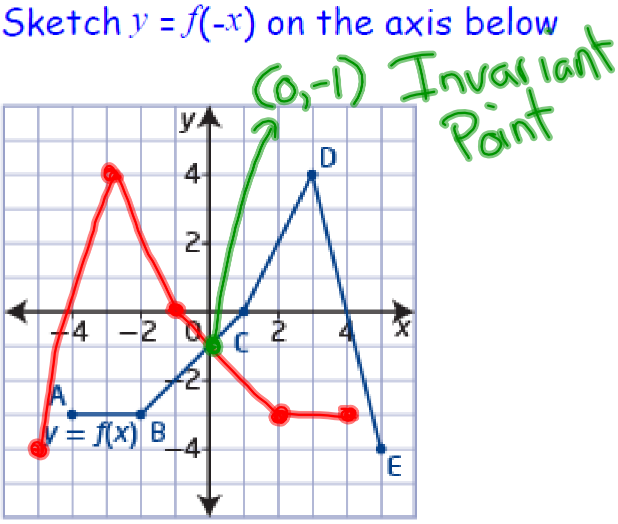
D $(3,4)$ $(3,-4)$

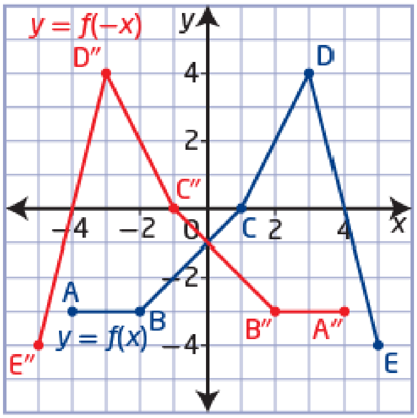
E $(5,-4)$ $(5,4)$

Remember...

- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

- Sketch $y = f(-x)$ on the axis below





$(x, y) \longrightarrow (-x, y)$

A $(-4,-3)$ $(4,-3)$

B $(-2,-3)$ $(2,-3)$

C $(1,0)$ $(-1,0)$

D $(3,4)$ $(-3,4)$

E $(5,-4)$ $(-5,-4)$

stretch + Compression

- a transformation in which the distance of each *x*-coordinate or *y*-coordinate from the line of reflection is multiplied by some scale factor
- scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection

Vertical and Horizontal Stretches

A stretch, unlike a translation or a reflection, changes the shape of the graph. However, like translations, stretches do not change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

Vertical Stretch $(x, y) \rightarrow (x, ay)$

① $y = 2f(x)$ $a=2$

$(x, y) \rightarrow (x, 2y)$

③ $y = \frac{1}{3}f(x)$ $a=\frac{1}{3}$

$(x, y) \rightarrow (x, \frac{1}{3}y)$

② $y = 3|x|$ $a=3$

$(x, y) \rightarrow (x, 3y)$

④ $y = -4x^2$ $a=4$

$(x, y) \rightarrow (x, -4y)$

reflection x axis

Horizontal Stretch: $(x, y) \rightarrow (\frac{1}{b}x, y)$

① $y = |2x|$ $b=2$

$(x, y) \rightarrow (\frac{1}{2}x, y)$

② $y = f(\frac{1}{3}x)$ $b=\frac{1}{3}$

$(x, y) \rightarrow (3x, y)$

Vertical Stretch or Compression...

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.

Example 2

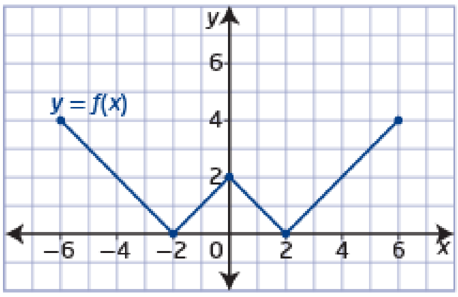
Graph $y = af(x)$

Given the graph of $y = f(x)$,

- transform the graph of $f(x)$ to sketch the graph of $g(x)$
- describe the transformation
- state any invariant points
- state the domain and range of the functions

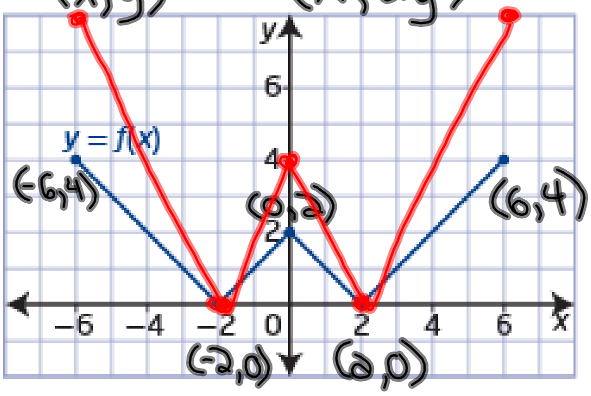
a) $g(x) = 2f(x)$

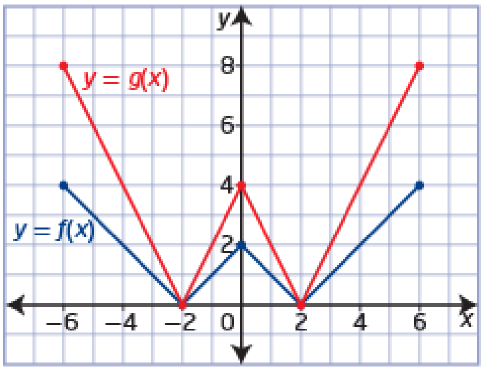
b) $g(x) = \frac{1}{2}f(x)$



a) $g(x) = 2f(x)$ Vertical Stretch $a=2$

$(x,y) \rightarrow (x, 2y)$





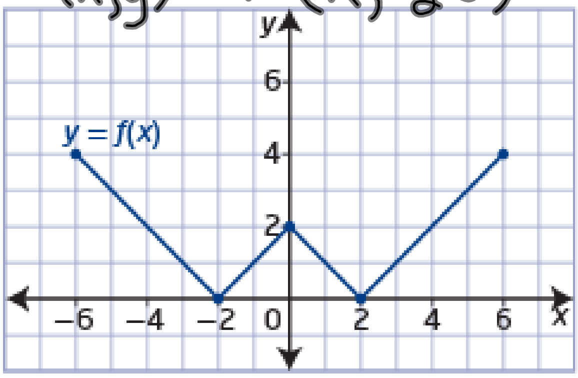
The invariant points are $(-2, 0)$ and $(2, 0)$.

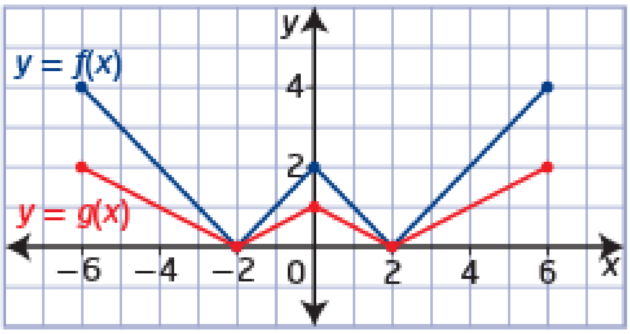
For $f(x)$, the domain is
$\{x \mid -6 \leq x \leq 6, x \in \mathrm{R}\}$, or $[-6, 6]$,
and the range is
$\{y \mid 0 \leq y \leq 4, y \in \mathrm{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in \mathrm{R}\}$, or $[-6, 6]$,
and the range is $\{y \mid 0 \leq y \leq 8, y \in \mathrm{R}\}$, or $[0, 8]$.

b) $g(x) = \frac{1}{2}f(x)$ $a = \frac{1}{2}$

$(x, y) \rightarrow (x, \frac{1}{2}y)$





The invariant points are (−2, 0) and (2, 0).

For $f(x)$, the domain is
$\{x \mid -6 \leq x \leq 6, x \in R\}$, or $[-6, 6]$,
and the range is
$\{y \mid 0 \leq y \leq 4, y \in R\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in R\}$, or $[-6, 6]$,
and the range is $\{y \mid 0 \leq y \leq 2, y \in R\}$, or $[0, 2]$.

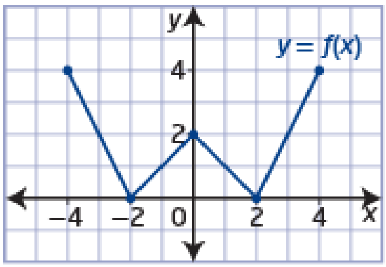
Horizontal Stretch or Compression...

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

Example 3

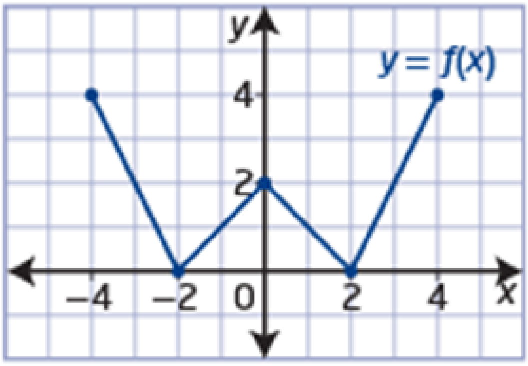
Graph $y = f(bx)$

Given the graph of $y = f(x)$,
- transform the graph of $f(x)$ to sketch the graph of $g(x)$
- describe the transformation
- state any invariant points
- state the domain and range of the functions



a) $g(x) = f(2x)$
b) $g(x) = f\left(\frac{1}{2}x\right)$

a) $g(x) = f(2x)$



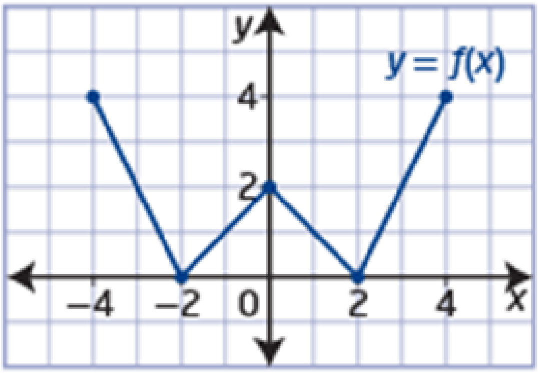
[redacted]

The invariant point is

For $f(x)$, the domain is
or and the range is
or

For $g(x)$, the domain is
or and the range is
or

b) $g(x) = f\left(\frac{1}{2}x\right)$



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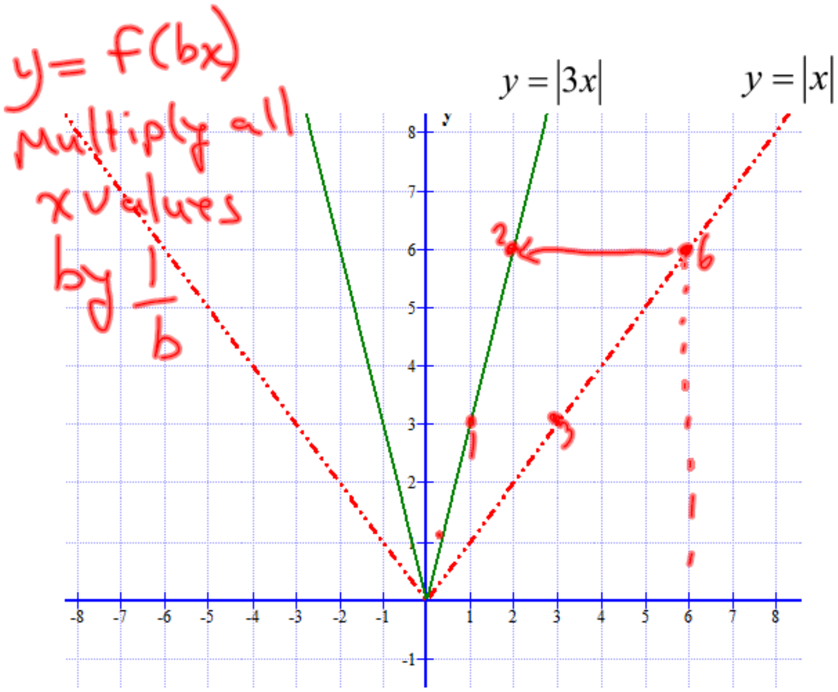
The invariant point is

For $f(x)$, the domain is
and the range is

For $g(x)$, the domain is
and the range is

Horizontal Stretch or Compression...



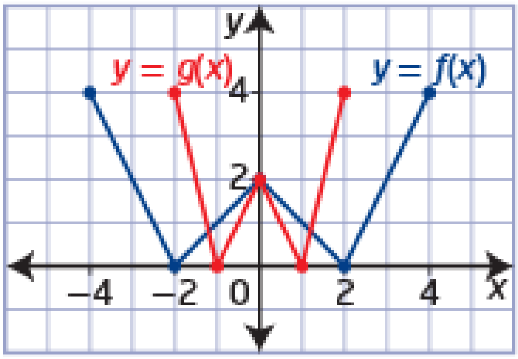
Horizontal Stretch or Compression...

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

$$y = -3f(-2x) + 7$$

Homework

Determine the Equation of a Translated Function:


y
y = g(x)
y = f(x)
4
2
−4
−2
0
2
4
x