

$$\textcircled{11} \text{ c) } \frac{xy(x-y)x}{y} - \frac{x+y}{x-y} \quad xy(x-y)$$

$$\frac{xy(x-y)y}{x} - \frac{x+y}{x-y} \quad xy(x-y)$$

$$\frac{x^2(x-y) - xy(x+y)}{y^2(x-y) - xy(x+y)}$$

$$\frac{x^3 - x^2y - x^2y - xy^2}{xy^2 - y^3 - x^2y - xy^2}$$

$$\frac{x^3 - 2x^2y - xy^2}{-y^3 - x^2y} \quad \text{Restrictions:}$$

$$x \neq 0, y$$

$$y \neq 0$$

$$\textcircled{18} \text{ a) } \frac{4}{(3x-4)} + \frac{3}{(x-1)} = 1 \quad (3x-4)(x-1)$$

$$4(x-1) + 3(3x-4) = (3x-4)(x-1)$$

$$4x - 4 + 9x - 12 = 3x^2 - 7x + 4$$

$$13x - 16 = 3x^2 - 7x + 4$$

$$0 = 3x^2 - 20x + 20$$

$$a = 3$$

$$b = -20$$

$$c = 20$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{20 \pm \sqrt{400 - 240}}{6}$$

$$x = \frac{20 \pm \sqrt{160}}{6} \quad \sqrt{16 \times 10} = 4\sqrt{10}$$

$$x = \frac{20 \pm 4\sqrt{10}}{6}$$

$$x = \frac{10 \pm 2\sqrt{10}}{3}$$

$$\textcircled{a} \text{ b) } \frac{x+6}{x^2-4} = \frac{a}{x-a} + \frac{x}{x+a}$$

$$\frac{\cancel{(x+a)(x-a)} x+6}{\cancel{(x+a)(x-a)}} = \frac{a}{\cancel{x-a}} + \frac{x}{\cancel{x+a}} \quad \cancel{(x+a)(x-a)}$$

$$x+6 = a(x+a) + x(x-a)$$

$$x+6 = ax+4 + x^2 - ax$$

$$x+6 = x^2 + 4$$

$$0 = x^2 - x - 2$$

$$0 = (x-2)(x+1)$$

$$x = -1, 2$$

check $(x=-1)$

check $(x=2)$

$$\textcircled{a} \text{c) } (\sqrt{3x+15})^2 = (1+\sqrt{18+x})^2$$

$$3x+15 = 1 + \sqrt{18+x} + \sqrt{18+x} + 18+x$$

$$3x+15 = \cancel{x} + 2\sqrt{18+x} + \cancel{19}$$

$$2x-4 = 2\sqrt{18+x}$$

$$2(x-2) = 2\sqrt{18+x}$$

$$(x-2)^2 = (\sqrt{18+x})^2$$

$$x^2 - 4x + 4 = 18 + x$$

$$x^2 - 5x - 14 = 0$$

$$(x-7)(x+2) = 0$$

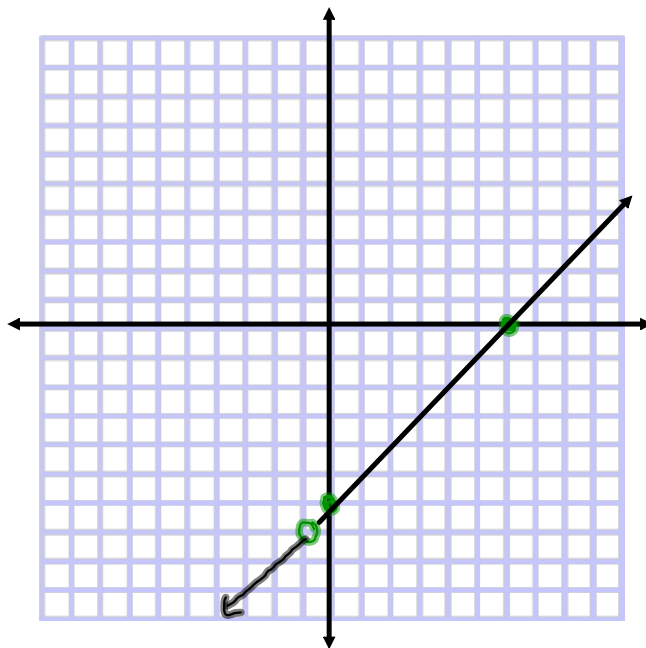
$$x = 7, -2$$

Check $(x=7)$

Check $(x=-2)$

$$\textcircled{4} \text{ a) } f(x) = \frac{x^2 - 5x - 6}{x+1} = \frac{(x-6)\cancel{(x+1)}}{\cancel{(x+1)}} = x-6$$

① Roots: $x=6$ ② V.A.: None ③ O.A.: $y=x-6$ ④ Holes: $x=-1$ ⑤ y int: $y=-6$



$$\textcircled{1} \text{ d) } \frac{\frac{2}{x} + \frac{3}{xy}}{\frac{2}{xy} + \frac{3}{y}} \rightarrow \frac{\frac{2y+3}{xy}}{\frac{2+3x}{xy}}$$

$$\rightarrow \frac{2y+3}{xy} \cdot \frac{xy}{2+3x} \rightarrow \boxed{\frac{2y+3}{2+3x}} \quad \begin{matrix} x \neq 0, -\frac{2}{3} \\ y \neq 0 \end{matrix}$$

$$\begin{aligned} 2+3x &= 0 \\ 3x &= -2 \\ x &= -\frac{2}{3} \end{aligned}$$

$$\frac{\overset{xy}{\frac{2}{x}} + \frac{3}{\overset{xy}{xy}}}{\overset{xy}{\frac{2}{xy}} + \frac{3}{\overset{xy}{y}}} \rightarrow \boxed{\frac{2y+3}{2+3x}} \quad \begin{matrix} x \neq 0, -\frac{2}{3} \\ y \neq 0 \end{matrix}$$

$$\textcircled{2} \text{ c) } (\sqrt{3x+15})^2 = (1 + \sqrt{18+x})^2$$

$$3x+15 = 1 + 2\sqrt{18+x} + 18+x$$

$$3x+15 = \cancel{19+x} + 2\sqrt{18+x}$$

$$2x-4 = 2\sqrt{18+x}$$

$$2(x-2) = 2\sqrt{18+x}$$

$$(x-2)^2 = (\sqrt{18+x})^2$$

$$x^2 - 4x + 4 = 18 + x$$

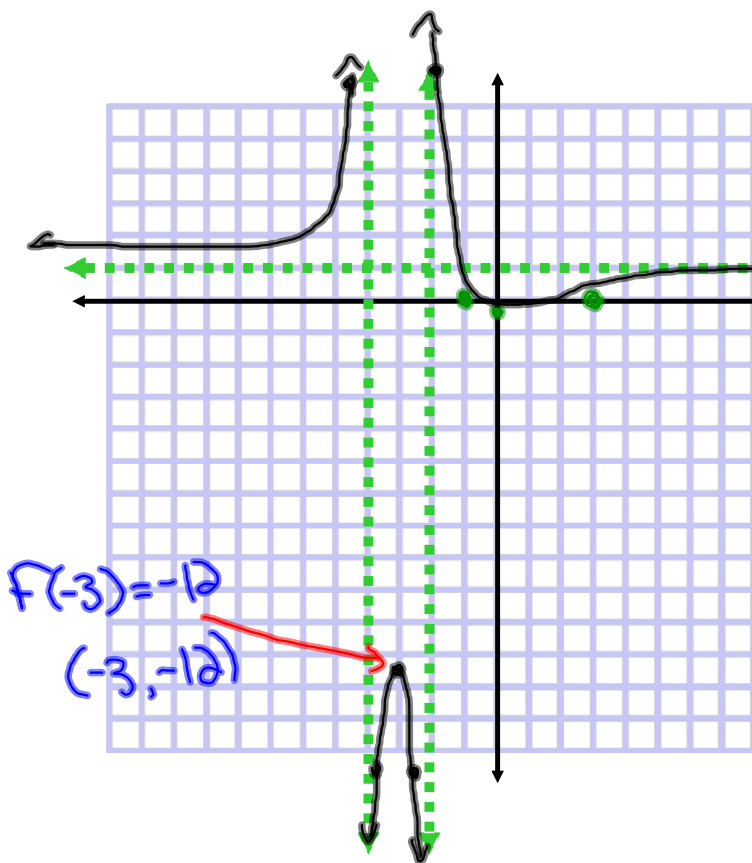
$$x^2 - 5x - 14 = 0$$

$$(x-7)(x+2) = 0$$

$$\boxed{x=7} \quad | \quad x=-2$$

$$\textcircled{4} \text{ b) } f(x) = \frac{x^2 - 2x - 3}{x^2 + 6x + 8} = \frac{(x-3)(x+1)}{(x+2)(x+4)}$$

① roots $x = -1, 3$ ② V.A. $x = -4, -2$ ③ H.A. $y = 1$ ④ Holes: None ⑤ y.int $y = -3/8$



Check Behaviour near V.A.

$$x = -4$$

$$\lim_{x \rightarrow -4^-} f(x) = +\infty$$

$$\lim_{x \rightarrow -4^+} f(x) = -\infty$$

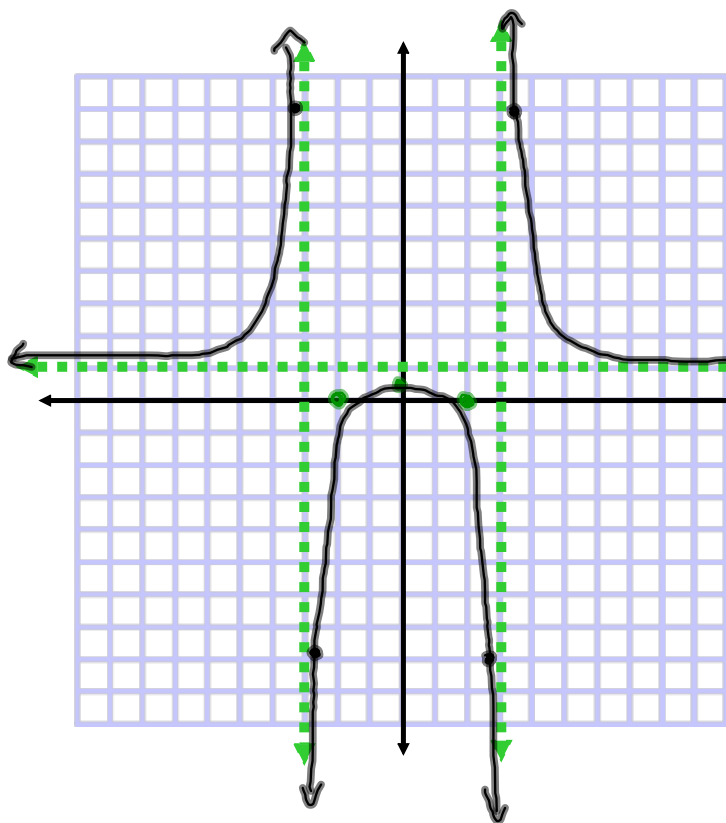
$$x = -2$$

$$\lim_{x \rightarrow -2^-} f(x) = -\infty$$

$$\lim_{x \rightarrow -2^+} f(x) = +\infty$$

$$\textcircled{4} \text{ c) } f(x) = \frac{x^2 - 4}{x^2 - 9} = \frac{(x+2)(x-2)}{(x+3)(x-3)}$$

- ① Roots: $x = \pm 2$ ② V.A. $x = \pm 3$ ③ H.A. $y = 1$ ④ Holes: None ⑤ y int $y = 4/9$



Check the behaviour near the V.A.

$$x = -3$$

$$\lim_{x \rightarrow -3^-} f(x) = +\infty$$

$$\lim_{x \rightarrow -3^+} f(x) = -\infty$$

$$x = 3$$

$$\lim_{x \rightarrow 3^-} f(x) = -\infty$$

$$\lim_{x \rightarrow 3^+} f(x) = +\infty$$

