

Ex. 2.6

$$\textcircled{6} \text{ n) } y = \sqrt{x + \sqrt{x + \sqrt{x}}} = \left( x + \left( x + x^{\frac{1}{2}} \right)^{\frac{1}{2}} \right)^{\frac{1}{2}}$$

$$y' = \frac{1}{2} \left( x + \left( x + x^{\frac{1}{2}} \right)^{\frac{1}{2}} \right)^{-\frac{1}{2}} \left[ 1 + \frac{1}{2} \left( x + x^{\frac{1}{2}} \right)^{-\frac{1}{2}} \left( 1 + \frac{1}{2} x^{-\frac{1}{2}} \right) \right]$$

$$y' = \left[ \frac{1}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \right] \left[ 1 + \left( \frac{1}{2\sqrt{x + \sqrt{x}}} \right) \left( 1 + \frac{1}{2\sqrt{x}} \right) \right]$$

$$y' = \left[ \frac{1}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \right] \left[ 1 + \left( \frac{1}{2\sqrt{x + \sqrt{x}}} \right) \left( \frac{2\sqrt{x} + 1}{2\sqrt{x}} \right) \right]$$

$$y' = \left[ \frac{1}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \right] \left[ 1 + \frac{2\sqrt{x} + 1}{4\sqrt{x}\sqrt{x + \sqrt{x}}} \right]$$

## Correct Homework Sheet

$$\textcircled{6} \text{ d) } G(x) = (x^4 - x + 1)^2 (x^2 - 2)^3$$

$$\begin{aligned} G'(x) &= (x^4 - x + 1)^2 (3)(x^2 - 2)^2 (2x) + 2(x^4 - x + 1)(4x^3 - 1)(x^2 - 2)^3 \\ &= 6x(x^4 - x + 1)^2 (x^2 - 2)^2 + 2(x^4 - x + 1)(x^2 - 2)^3 (4x^3 - 1) \\ &= 2(x^4 - x + 1)(x^2 - 2)^2 \left[ 3x(x^4 - x + 1) + (x^2 - 2)(4x^3 - 1) \right] \\ &= 2(x^4 - x + 1)(x^2 - 2)^2 (7x^5 - 8x^3 - 4x^2 + 3x + 2) \end{aligned}$$

$$\textcircled{7} \quad y - y_1 = m(x - x_1)$$

$$\text{Point: } \left( 2, \frac{1}{2} \right) \quad \text{(i) } y = \frac{1}{\sqrt{20-x^4}} = \frac{1}{(20-x^4)^{1/2}} = (20-x^4)^{-1/2}$$

$$m = \text{slope} = y'(2) \quad y' = -\frac{1}{2}(20-x^4)^{-3/2}(-4x^3)$$

$$\text{(ii) } y'(2) = \frac{2(2)^3}{\sqrt{(20-2^4)^3}}$$

$$y' = \frac{2x^3}{(20-x^4)^{3/2}}$$

$$= \frac{16}{\sqrt{64}}$$

$$y' = \frac{2x^3}{\sqrt{(20-x^4)^3}}$$

$$\boxed{m = 2}$$

$$\text{(iii) } y - y_1 = m(x - x_1)$$

$$y - \frac{1}{2} = 2(x - 2)$$

$$y - \frac{1}{2} = 2x - 4 \quad \rightarrow \quad 2x - y - \frac{7}{2} = 0$$

$$2y - 1 = 4x - 8$$

$$\boxed{0 = 4x - 2y - 7}$$

$$\textcircled{9} \quad g(2) = 4$$

$$F(x) = f(g(x))$$

$$g'(2) = 3$$

$$F'(x) = f'(g(x))g'(x)$$

$$f'(4) = 5$$

$$F'(2) = f'(g(2))g'(2)$$

$$F'(2) = ?$$

$$= [f'(4)][g'(2)]$$

$$= (5)(3)$$

$$= 15$$

## Correct Homework Sheet

$$\textcircled{2} \text{ b) } y = \sqrt[3]{\frac{1-x^6}{2+(5x-1)^4}} = \left[ \frac{1-x^6}{2+(5x-1)^4} \right]^{\frac{1}{3}} .$$

$$y' = \frac{1}{3} \left[ \frac{1-x^6}{2+(5x-1)^4} \right]^{-\frac{2}{3}} \left[ \frac{(2+(5x-1)^4)(-6x^5) - (1-x^6)(4)(5x-1)^3(5)}{[2+(5x-1)^4]^2} \right]$$

## Correct Homework Sheet

$$\textcircled{3} b) f(x) = \frac{8x^3(12x^2-5x)^8}{2-3(1-32x^{10})^{1/5}}$$

$$\frac{[2-3(1-32x^{10})^{1/5}] [(8x^2)(8)(12x^2-5x)(24x-5) + (24x^3)(12x^2-5x)^7] - [8x^3(12x^2-5x)^8] [(-\frac{3}{5})(1-32x^{10})^{-4/5}(-320x^9)]}{[2-3(1-32x^{10})^{1/5}]^2}$$

$$\textcircled{3} c) f(x) = \frac{[x^5 - x(4-x^2)^{1/2}]^6}{12x^{1/2}(5x^3-8)^7}$$

$$f'(x) = \frac{[12x^{1/2}(5x^3-8)^7] [6[x^5 - x(4-x^2)^{1/2}]^5 [5x^4 - [x(\frac{1}{2})(4-x^2)^{-1/2}(-2x) + (4-x^2)^{1/2}]] - [x^5 - x(4-x^2)^{1/2}]^6 [(12x^{1/2})(7)(5x^3-8)^6(15x^2) + (6x^{-1/2})(5x^3-8)^7]}{[12x^{1/2}(5x^3-8)^7]^2}$$

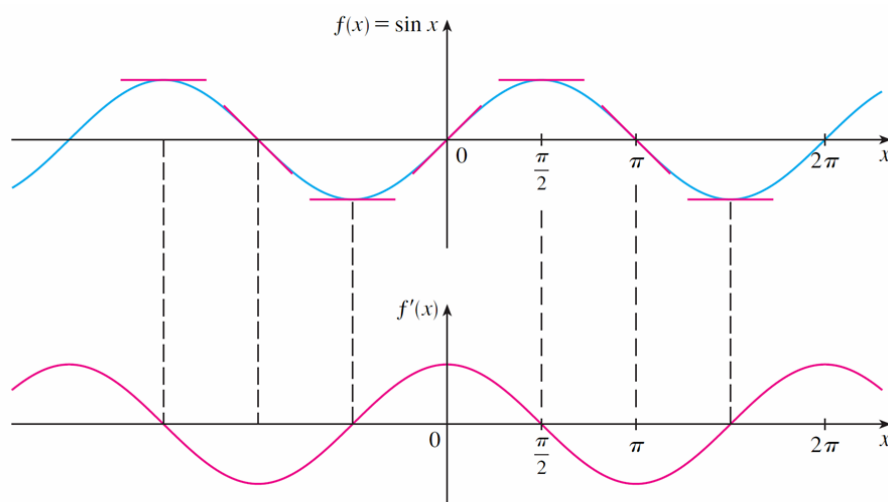
To be handed in today  
Differentiate the following (do not simplify)



## Derivatives of Trigonometric Functions

### The Sine Function

- We recall that the derivative  $f'(x)$  of a function  $f(x)$  gives the slope of the tangent.
- On the next slide we graph  $f(x) = \sin x$  together with  $f'(x)$ , as determined by the slope of the tangent to the sine curve.
  - Note that  $x$  is measured in radians.
- The derivative graph resembles the graph of the cosine!



Let's check this using the definition of a derivative...

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h} \\
 &= \lim_{h \rightarrow 0} \left[ \frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right] \\
 &= \lim_{h \rightarrow 0} \left[ \frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right] \\
 &= \lim_{h \rightarrow 0} \left[ \sin x \left( \frac{\cos h - 1}{h} \right) + \cos x \left( \frac{\sin h}{h} \right) \right] \\
 &= \lim_{h \rightarrow 0} \sin x \cdot \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} + \lim_{h \rightarrow 0} \cos x \cdot \lim_{h \rightarrow 0} \frac{\sin h}{h}
 \end{aligned}$$

■ Our calculations have brought us to four limits, two of which are easy:

■ Since  $x$  is constant while  $h \rightarrow 0$ ,

$$\lim_{h \rightarrow 0} \sin x = \sin x \quad \text{and} \quad \lim_{h \rightarrow 0} \cos x = \cos x$$

■ With some work we can also show that

$$\lim_{h \rightarrow 0} \frac{\sin h}{h} = 1 \quad \text{and} \quad \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} = 0$$

■ Thus our guess is confirmed:

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \sin x \cdot \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} + \lim_{h \rightarrow 0} \cos x \cdot \lim_{h \rightarrow 0} \frac{\sin h}{h} \\
 &= (\sin x) \cdot 0 + (\cos x) \cdot 1 = \cos x
 \end{aligned}$$

## Rules to differentiate trigonometric functions:

Given that "u" represents some differentiable function...

$$\frac{d}{du}(\sin u) = \cos u \bullet du$$

$$\frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$$

$$\frac{d}{du}(\cos u) = -\sin u \bullet du$$

$$\frac{d}{du}(\sec u) = \sec u \tan u \bullet du$$

$$\frac{d}{du}(\tan u) = \sec^2 u \bullet du$$

$$\frac{d}{du}(\cot u) = -\csc^2 u \bullet du$$



## Let's Practice...

Differentiate the following:

$$y = \sin 3x$$

$$y = \sin(x + 2)$$

$$y = \sin(kx + d)$$

**Ex #2.**

Differentiate:

a)  $y = \sin(x^3)$

b)  $y = \sin^3 x$

c)  $y = \sin^3(x^2 - 1)$

Ex #3.

Differentiate:

$$y = x^2 \cos x$$

# Homework

 [Worksheet on derivatives of trigonometric functions](#)

## Attachments

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Derivatives Worksheet.doc