

Using Factor Theorem

③ a) $x^3 - x^2 - 14x + 24 \rightarrow x-2$ is a factor

$$\begin{aligned} & (2)^3 - (2)^2 - 14(2) + 24 \\ & 8 - 4 - 28 + 24 \\ & 0 \end{aligned}$$

$$\begin{array}{r|l} x-2 & \begin{array}{l} x^2 + x - 12 \\ \underline{-(x^3 - 2x^2)} \\ x^2 - 14x + 24 \\ \underline{-(x^2 - 2x)} \\ -12x + 24 \\ \underline{-(-12x + 24)} \\ 0 \end{array} \\ \hline & \begin{array}{l} (x-2)(x^2+x-12) \\ (x-2)(x-3)(x+4) \end{array} \end{array}$$

simple trinomial

Using Synthetic Substitution:

$x^3 - x^2 - 14x + 24 \rightarrow x-2$ is a factor

$$\begin{aligned} & (2)^3 - (2)^2 - 14(2) + 24 \\ & 8 - 4 - 28 + 24 \\ & 0 \end{aligned}$$

x-value	coefficients			
2	1	-1	-14	24
		2	2	-24
	1	1	-12	

$$\begin{array}{l} (x-2)(x^2+x-12) \\ \boxed{(x-2)(x-3)(x+4)} \end{array}$$

29) b) $x^3 - 3x^2 - 33x + 35$ $(x-1)$ is a factor
 $(1)^3 - 3(1)^2 - 33(1) + 35$
 $1 - 3 - 33 + 35$
 0

31) c) $x^3 + 4x^2 - x - 10$ Factor Theorem

$x-3$ $\overline{) x^4 + x^3 - 13x^2 - 7x + 30}$

$-(x^4 - 3x^3)$ ↓

$4x^3 - 13x^2$

$-(4x^3 - 12x^2)$ ↓

$-x^2 - 7x$

$-(-x^2 + 3x)$ ↓

$-10x + 30$

$-(-10x + 30)$

0

$x+2$ $\overline{) x^3 + 4x^2 - x - 10}$

$-(x^3 + 2x^2)$ ↓

$2x^2 - x$

$-(2x^2 + 4x)$ ↓

$-5x - 10$

$-(5x + 10)$

0

Factors:

$(x-3)(x+2)(x^2+2x-5)$

32 a)

(i)
x-value

2

$$\begin{array}{r} 1 \quad -4 \quad 1 \quad 6 \\ \quad 2 \quad -4 \quad -6 \\ \hline 1 \quad -2 \quad -3 \end{array}$$

$$(a-2)(a^2-2a-3)$$

$$(a-2)(a-3)(a+1)$$

Factor Theorem

③ a) $x^3 - x^2 - 14x + 24$ $(x-2)$ is a factor

$(2)^3 - (2)^2 - 14(2) + 24$

$8 - 4 - 28 + 24$

0

$$\begin{array}{r}
 x-2 \overline{) x^3 - x^2 - 14x + 24} \\
 \underline{-(x^3 - 2x^2)} \\
 x^2 - 14x \\
 \underline{-(x^2 - 2x)} \\
 -12x + 24 \\
 \underline{-(-12x + 24)} \\
 0
 \end{array}$$

Simple Trinomial

$(x-2)(x^2+x-12)$
 $(x-2)(x-3)(x+4)$

Synthetic Substitution

③ a) $x^3 - x^2 - 14x + 24$

$(2)^3 - (2)^2 - 14(2) + 24$

$8 - 4 - 28 + 24$

0

x value	coefficients			
2	1	-1	-14	24
		2	2	-24
	<hr/>			
	1	1	-12	

$(x-2)(x^2+x-12)$
 $(x-2)(x-3)(x+4)$

33) a) $x^4 - 3x^3 - 5x^2 + 3x + 4 = 0$

$x=1$ $1 - 3 - 5 + 3 + 4 = 0$

$$\begin{array}{r|rrrrr} 1 & 1 & -3 & -5 & 3 & 4 \\ & & 1 & -2 & -7 & -4 \\ \hline & 1 & -2 & -7 & -4 & \end{array}$$

$(x-1)(x^3 - 2x^2 - 7x - 4)$
 $-1 -2 +7 -4$

Synthetic Substitution

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -7 & -4 \\ & & -1 & 3 & 4 \\ \hline & 1 & -3 & -4 & \end{array}$$

$(x-1)(x+1)(x^2 - 3x - 4)$

Simple Trinomial

$-4 \times 1 = -4$
 $-4 + 1 = -3$

$(x-1)(x+1)(x-4)(x+1)$

$(x-4)(x-1)(x+1)^2$

$$\textcircled{31} \text{ e) } x^3 - 27x + 10 \div x - 5$$

$$\begin{array}{r}
 \underline{x} - 5 \overline{) x^3 + 0x^2 - 27x + 10} \\
 \underline{-(x^3 - 5x^2)} \\
 5x^2 - 27x \\
 \underline{-(5x^2 - 25x)} \\
 -2x + 10 \\
 \underline{-(-2x + 10)} \\
 0
 \end{array}$$

$$(x-5)(x^2+5x-2)$$

$$\textcircled{33} \quad x^4 - 3x^3 - 5x^2 + 3x + 4 = 0$$

$$(1)^4 - 3(1)^3 - 5(1)^2 + 3(1) + 4 = 0$$

$$1 - 3 - 5 + 3 + 4 = 0$$

$$0 = 0$$

$$\begin{array}{r|rrrrr} 1 & 1 & -3 & -5 & 3 & 4 \\ & & 1 & -2 & -7 & -4 \\ \hline & 1 & -2 & -7 & -4 & 0 \end{array}$$

$$(x-1)(x^3 - 2x^2 - 7x - 4) \leftarrow \text{Synthetic Sub.}$$

$$(-1)^3 - 2(-1)^2 - 7(-1) - 4$$

$$-1 - 2 + 7 - 4 = 0$$

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -7 & -4 \\ & & -1 & 3 & 4 \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

$$(x-1)(x+1)(x^2 - 3x - 4) \leftarrow \text{Simple Trinomial}$$

$$(x-1)(x+1)(x-4)(x+1)$$

$$\text{or } (x-4)(x-1)(x+1)^2$$

Advanced Math

31) c)

$$\begin{array}{r} \underline{\underline{x-3}} \overline{) x^4 + x^3 - 13x^2 - 7x + 30} \\ \underline{-(x^4 - 3x^3)} \\ 4x^3 - 13x^2 \\ \underline{-(4x^3 - 12x^2)} \\ -x^2 - 7x + 30 \\ \underline{-(-x^2 + 3x)} \\ -10x + 30 \\ \underline{-(-10x + 30)} \\ 0 \end{array}$$

$(x-3)(x^3 + 4x^2 - x - 10)$

Factor further
(Factor Theorem)

$(-2)^3 + 4(-2)^2 - (-2) - 10$
 $-8 + 16 + 2 - 10$
 0

$(x+2)$ is a factor

$$\begin{array}{r} \underline{\underline{x+2}} \overline{) x^3 + 4x^2 - x - 10} \\ \underline{-(x^3 + 2x^2)} \\ 2x^2 - x \\ \underline{-(2x^2 + 4x)} \\ -5x - 10 \\ \underline{-(-5x - 10)} \\ 0 \end{array}$$

$(x-3)(x+2)(x^2 + 2x - 5)$

$(x-3)(x+2)(x^2 + 2x - 5) = x^4 + x^3 - 13x^2 - 7x + 30$
 "factored form"

$$\begin{array}{r}
 \textcircled{31} \quad f) \quad \frac{3x^4 + 7x^3 + 5x^2 + 7x + 2}{x-1} \\
 \hline
 \underline{-(3x^5 - 3x^4)} \quad \downarrow \\
 7x^4 - 2x^3 \quad \downarrow \\
 \underline{-(7x^4 - 7x^3)} \quad \downarrow \\
 5x^3 + 2x^2 \quad \downarrow \\
 \underline{-(5x^3 - 5x^2)} \quad \downarrow \\
 7x^2 - 5x \quad \downarrow \\
 \underline{-(7x^2 - 7x)} \quad \downarrow \\
 2x - 2 \quad \downarrow \\
 \underline{-(2x - 2)} \\
 0
 \end{array}$$

$(x-1)(3x^4 + 7x^3 + 5x^2 + 7x + 2)$ Factor further "Factor Theorem"
 $3(-2)^4 + 7(-2)^3 + 5(-2)^2 + 7(-2) + 2$
 $48 - 56 + 20 - 14 + 2$
 0 $(x+2)$ is a factor

$$\begin{array}{r}
 \frac{3x^3 + x^2 + 3x + 1}{x+2} \\
 \hline
 \underline{-(3x^4 + 6x^3)} \quad \downarrow \\
 x^3 + 5x^2 \quad \downarrow \\
 \underline{-(x^3 + 2x^2)} \quad \downarrow \\
 3x^2 + 7x \quad \downarrow \\
 \underline{-(3x^2 + 6x)} \quad \downarrow \\
 x + 2 \quad \downarrow \\
 \underline{-(x + 2)} \\
 0
 \end{array}$$

$(x-1)(x+2)(3x^3 + x^2 + 3x + 1)$ Group for a common factor

$$(x-1)(x+2)(x^2(3x+1) + 1(3x+1))$$

$$(x-1)(x+2)(3x+1)(x^2+1)$$

$$\textcircled{32} \text{ a) (i) } m^3 - 7m + 6 \div m - 1$$

Synthetic Substitution

m value

coefficients

1

1 0 -7 6

1 1 -6

1 1 -6 0

"coefficients of other factor"

$$(m-1)(m^2+m-6)$$

$$(m-1)(m-2)(m+3)$$

Difference of Cubes

$$\left. \begin{array}{l} (a^3 - b^3) \rightarrow (a-b)(a^2 + ab + b^2) \\ (8x^3 - 125) \rightarrow (2x-5)(4x^2 + 10x + 25) \end{array} \right\}$$

Sum of Cubes

$$(a^3 + b^3) \rightarrow (a+b)(a^2 - ab + b^2)$$

Adv Math Quiz

- Factoring
- Composite Functions
- Sketching Functions
 - ↳ Roots (x int)
 - ↳ y int
 - ↳ degree
 - ↳ stretch factor
 - ↳ local max/min

Find missing term

$$\textcircled{1} f) \quad x^4 + \underline{5}x^2 + 49 \quad \begin{array}{l} \sqrt{1 \cdot 49} = 7 \\ 7 \times 2 = 14 \end{array}$$

$$(x^4 + 14x^2 + 49) - 9x^2$$

$$(x^2 + 7)^2 - 9x^2$$

$$(x^2 + 7 - 3x)(x^2 + 7 + 3x)$$

$$(x^2 - 3x + 7)(x^2 + 3x + 7)$$

$$\textcircled{1} \text{ e) } x^4 + 7x^2 + 12$$

$$(x^2 + 3)(x^2 + 4)$$

$$\text{f) } x^4 + \underline{5}x^2 + 49 \quad \sqrt{49} = 7 \cdot 2 = 14$$

$$(x^4 + 14x^2 + 49) - 9x^2$$

$$\boxed{(x^2 + 7)^2} - \boxed{9x^2}$$

$$((x^2 + 7) + 3x)((x^2 + 7) - 3x) \checkmark$$

$$(x^2 + 3x + 7)(x^2 - 3x + 7)$$

$$\textcircled{1} \text{ g) } \underline{49}x^2 - 84x + \underline{36}$$
$$(7x - 6)(7x - 6)$$
$$(7x - 6)^2$$

you could
use decomp.

Adv. Math

$$\begin{aligned}
 \textcircled{1} \text{ i) } & 9x^2 - 30x - a^2 + 25 \\
 & (9x^2 - 30x + 25) - a^2 \\
 & (3x - 5)(3x - 5) - a^2 \\
 & \boxed{(3x - 5)^2} - \boxed{a^2} \\
 & ((3x - 5) + a)((3x - 5) - a)
 \end{aligned}$$

$$\textcircled{a} \ a) \quad x^3 + 2x^2 - 11x - 12$$

$$(-1)^3 + 2(-1)^2 - 11(-1) - 12$$

$$-1 + 2 + 11 - 12$$

$$0$$

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -11 & -12 \\ & & -1 & -1 & 12 \\ \hline & 1 & 1 & -12 & \emptyset \end{array}$$

$$(x+1)(x^2 + x - 12)$$

$$(x+1)(x+4)(x-3)$$

Adv Math:

$$\textcircled{4} \quad f(x) = x^2 + 3 \quad g(x) = 2x - 3$$

a) $f(g(x))$

$$\begin{aligned} f(2x-3) &= (2x-3)^2 + 3 \\ &= 4x^2 - 12x + 9 + 3 \\ &= 4x^2 - 12x + 12 \end{aligned}$$

$$\textcircled{1} a) \quad 8x^3 - 125$$
$$(2x - 5)(4x^2 + 10x + 25)$$

$$h) \quad 5a^3 - 10a^2 - 40a$$
$$5a(a^2 - 2a - 8)$$
$$(5a)(a - 4)(a + 2)$$

$$\begin{aligned} \textcircled{f}) \quad & x^4 + \underline{5}x^2 + 49 && \sqrt{49} = 7 \cdot 2 = 14 \\ & (x^4 + 14x^2 + 49) - 9x^2 \\ & \boxed{(x^2 + 7)^2} - \boxed{9x^2} \\ & (x^2 + 7 + 3x)(x^2 + 7 - 3x) \end{aligned}$$

$$\begin{aligned} \textcircled{2} \text{ a) } & \cdot x^3 + 2x^2 - 11x - 12 \\ & (-1)^3 + 2(-1)^2 - 11(-1) - 12 \\ & -1 + 2 + 11 - 12 \\ & 0 \end{aligned}$$

$$x = -1$$

$(x+1)$ is a factor

$$\begin{array}{r} \underline{-1} \Big| \quad 1 \quad 2 \quad -11 \quad -12 \\ \quad \quad -1 \quad -1 \quad 12 \\ \hline \quad 1 \quad 1 \quad -12 \end{array}$$

$$(x+1)(x^2 + x - 12)$$

$$(x+1)(x+4)(x-3)$$