Questions from Homework

m) $y = (1+\cos^2 x)^6 = (1+(\cos x)^2)^6$

$y' = 6(1+(\cos x)^2)^5 \cdot 2(\cos x)(-\sin x)$

$y' = 6(1+\cos^2 x)^5(-2\sin x\cos x)$

$y' = -6(1+\cos^2 x)^5(2\sin x\cos x)$ Double Angle Identity

$y' = -6(1+\cos^2 x)^5(\sin 2x)$

$$y' = -6\sin 2x(1+\cos^2 x)^5$$

b) $y = \dfrac{\sin x}{1+\cos x}$

$y' = \dfrac{(1+\cos x)(\cos x)(1) - \sin x(-\sin x)(1)}{(1+\cos x)^2}$

$y' = \dfrac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$ Pythagorean Identity

$y' = \dfrac{1+\cos x}{(1+\cos x)^2} = \boxed{\dfrac{1}{1+\cos x}}$

g) $y = \sin^{-2}(x^3) = (\sin(x^3))^{-2}$

$$y' = -2(\sin(x^3))^{-3} \cdot \cos(x^3) \cdot 3x^2$$

$$y' = \frac{-6x^2\cos(x^3)}{(\sin(x^3))^3} = \boxed{\frac{-6x^2\cos(x^3)}{\sin^3(x^3)}}$$

p) $y = \cos^3(\sin x) = [\cos(\sin x)]^3$

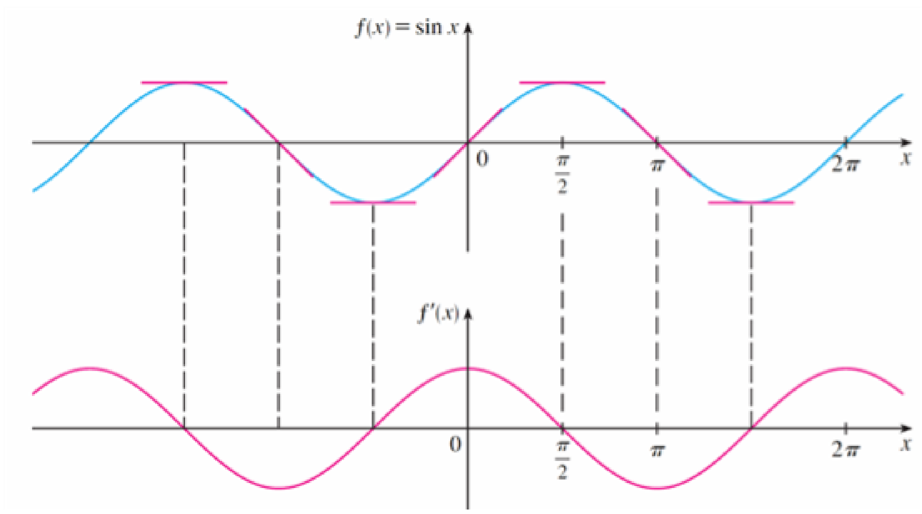
$$y' = 3[\cos(\sin x)]^2 \cdot (-\sin(\sin x)) \cdot (\cos x)$$

$$y' = -3[\cos(\sin x)]^2 \sin(\sin x)\cos x$$

$$\boxed{y' = -3\cos^2(\sin x)\sin(\sin x)\cos x}$$

The Sine Function

- We recall that the derivative $f'(x)$ of a function $f(x)$ gives the slope of the tangent.
- On the next slide we graph $f(x) = \sin x$ together with $f'(x)$, as determined by the slope of the tangent to the sine curve.
  - Note that x is measured in radians.
- The derivative graph resembles the graph of the cosine!



Let's check this using the definition of a derivative...

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) \right]$$

$$= \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

- Our calculations have brought us to four limits, two of which are easy:
  - Since x is constant while $h \to 0$,

$$\lim_{h\to 0} \sin x = \sin x \quad \text{and} \quad \lim_{h\to 0} \cos x = \cos x$$

- With some work we can also show that

$$\lim_{h\to 0} \frac{\sin h}{h} = 1 \quad \text{and} \quad \lim_{h\to 0} \frac{\cos h - 1}{h} = 0$$

- Thus our guess is confirmed:

$$f'(x) = \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

$$= (\sin x) \cdot 0 + (\cos x) \cdot 1 = \cos x$$

Rules to differentiate trigonometric functions:

Given that "u" represents some differentiable function...

$$\frac{d}{du}(\sin u) = \cos u \bullet du$$

$$\frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$$

$$\frac{d}{du}(\cos u) = -\sin u \bullet du$$

$$\frac{d}{du}(\sec u) = \sec u \tan u \bullet du$$

$$\frac{d}{du}(\tan u) = \sec^2 u \bullet du$$

$$\frac{d}{du}(\cot u) = -\csc^2 u \bullet du$$

Let's Practice...

Differentiate the following:

$$f(x)=\frac{1}{1+\tan x}$$

$$f'(x)=\frac{(1+\tan x)(0)-1(\sec^2 x)(1)}{(1+\tan x)^2}$$

$$=\frac{-\sec^2 x}{(1+\tan x)^2}$$

$$f(x)=\frac{1}{1+\tan x}=(1+\tan x)^{-1}$$

$$f'(x)=-(1+\tan x)^{-2}\cdot(\sec^2 x)(1)$$

$$=\frac{-\sec^2 x}{(1+\tan x)^2}$$

Ex #2.

Differentiate:

$$f(x) = 2\csc^3(3x^2) = 2(\csc(3x^2))^3$$

$$f'(x) = 6(\csc(3x^2))^2 \cdot -\csc(3x^2)\cot(3x^2) \cdot 6x$$

$$= -36x\csc^2(3x^2)\csc(3x^2)\cot(3x^2)$$

$$\boxed{= -36x\csc^3(3x^2)\cot(3x^2)}$$

Homework

Worksheet on derivatives of trigonometric functions

Attachments

Derivatives Worksheet.doc