Solving Polynomial Inequalities

Using the Graph

where does the function have positive "y" values

A polynomial inequality, $x^3 + x^2 - 9x - 9 > 0$, can be solved by examining the graph of the corresponding polynomial function, $y = (x^3 + x^2)(-9x - 9)$

- Roots: $y = 0$

$y = x^2(x+1) - 9(x+1)$

$y = (x+1)(x^2 - 9)$

$y = (x+1)(x+3)(x-3)$

$\boxed{x = -3, -1, 3}$

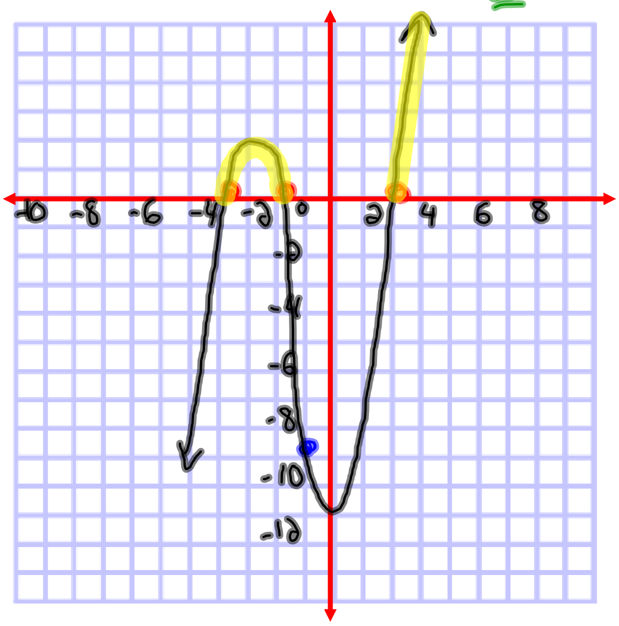
- y intercept $(x=0)$

$y = x^3 + x^2 - 9x - 9$

$y = (0)^3 + (0)^2 - 9(0) - 9$

$\boxed{y = -9}$

- Degree $\rightarrow$ 3rd
- Stretch Factor: $a = 1$



$\boxed{-3 < x < -1 \text{ and } x > 3}$

$x \in (-3, -1)$ $\quad$ $x \in (3, \infty)$

$\boxed{x \in (-3, -1) \cup (3, \infty)}$

Interval Notation

The statement ***-2 < x < 3*** can be written as $x \in (-2, 3)$; that is x belongs to the interval ***(-2, 3).*** The round brackets mean that x is not equal to ***-2 or 3.***

The statement -4 $\le x \le 2$ can be written as $x \in [-4, 2]$. The square brackets mean that x may be equal to ***-4 or 2.***

Explain the meaning of the following interval notations.

$x \in (-\infty, 2)$	$-\infty < x < 2$	$x < 2$
$x \in (-\infty, 2]$	$-\infty < x \le 2$	$x \le 2$
$x \in (3, \infty)$	$3 < x < \infty$	$x > 3$
$x \in [3, \infty)$	$3 \le x < \infty$	$x \ge 3$

Note: Infinity cannot be inclusive

Solving Polynomial Inequalities

Using the Number Line

Example: $x^3 + x^2 > 6x$

Step 1: State the Roots of the function

Step 2: Draw a number line and mark the roots of the equation. These roots separate the rest of the number line into three intervals.

$x \in (-\infty, \text{small x-int})$

$x \in (\text{small x-int, large x-int})$

$x \in (\text{large x-int}, \infty)$

Step 3: The value of the expression $x^3 + x^2 - 6x$ has the same sign throughout each interval in step 2 **because a function can only change signs at a root.** Therefore, choose a *test value of x* in each interval and evaluate the expression. Write a *plus or a minus* over that interval on the number line to indicate whether the expression is positive or negative.

Step 4: State the intervals for which $x^3 + x^2 - 6x > 0$

Solve using a number line

$$x^3 + x^2 > 6x$$

$$x^3 + x^2 - 6x > 0$$ positive y values

① Roots

$$y = x^3 + x^2 - 6x$$

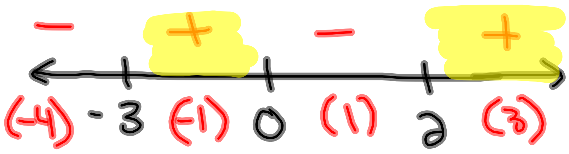
$$y = x(x^2 + x - 6)$$

$$y = x(x+3)(x-2)$$

$$0 = (x)(x+3)(x-2)$$

$$x = -3, 0, 2$$

②/③



④ $x \in (-3, 0) \cup (2, \infty)$

Using the Number Line

Example: $x^3 + x^2 > 6x$

$$\boxed{x^3 + x^2 - 6x > 0}$$

"+" y values

Step 1: State the Roots of the function

$y = x^3 + x^2 - 6x$

$y = x(x^2 + x - 6)$

$y = x(x+3)(x-2)$

Roots:

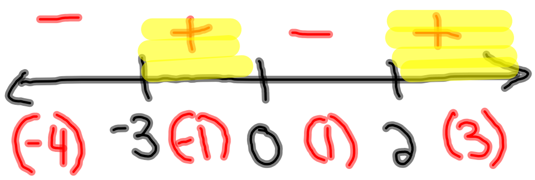
$$\boxed{x = -3, 0, 2}$$

Step 2: Draw a number line and mark the roots of the equation. These roots separate the rest of the number line into three intervals.

x $\in$ ***(-∞ , small x-int)***

x $\in$ ***(small x-int, large x-int)***

x $\in$ ***(large x-int, ∞)***



Step 3: The value of the expression $x^3 + x^2 - 6x$ has the same sign throughout each interval in step 2 **because a function can only change signs at a root.** Therefore, choose a *test value of x* in each interval and evaluate the expression. Write a *plus or a minus* over that interval on the number line to indicate whether the expression is positive or negative.

$x = -4$

$y = x(x+3)(x-2)$

$y = (-4)(-1)(-6)$

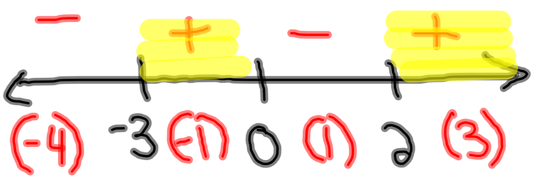
$y = -24$

$x = -1$

$y = (-)(+)(-)$

$y = +$

Step 4: State the intervals for which $x^3 + x^2 - 6x > 0$



$x \in (-3, 0) \cup (2, \infty)$

Homework

④ c) $2x^2 \geq 9x - 9$

$2x^2 - 9x + 9 \boxed{\geq} 0$

y values are greater than or equal to 0

$y = 2x^2 - 9x + 9$ ← Trinomial Decomp.

$-\underline{6} \times \underline{-3} = 18$

$-\underline{6} + \underline{-3} = -9$

$y = (2x^2 - 6x)(-3x + 9)$

$y = 2x(x-3) - 3(x-3)$

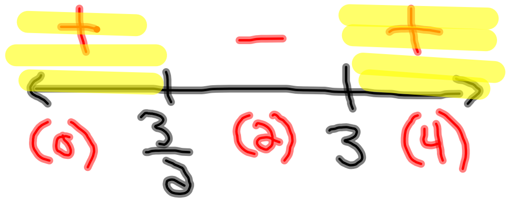
$y = (2x-3)(x-3)$

① Roots: $2x - 3 = 0$ | $x - 3 = 0$

$2x = 3$ | $x = 3$

$x = 3/2$

②/③



④ $x \in (-\infty, 3/2] \cup [3, \infty)$

⑦ b) $x^3-4x^2-x+4<0$ y values are negative

$y=(x^3-4x^2)(-x+4)$

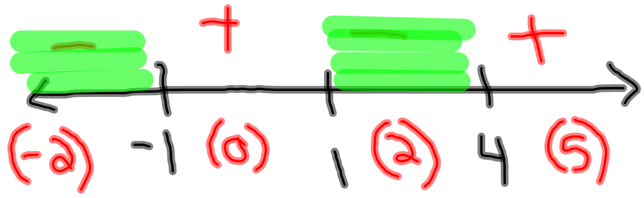
$y=x^2(x-4)-1(x-4)$

$y=(x^2-1)(x-4)$

$y=(x+1)(x-1)(x-4)$

① Roots: $x=-1,1,4$

②/③



④ $x\in(-\infty,-1)\cup(1,4)$

$x^3+x^2-9x-9 > 0$ ← positive y values

$y = x^3+x^2-9x-9$

① $y=(x^3+x^2)(-9x-9)$

$y = x^2(x+1)-9(x+1)$

$(-)(-)(-)$ $y=(x+1)(x^2-9)$

$(-)(+)(-)$ $y=(x+1)(x+3)(x-3)$

$(+)(+)(-)$

$(+)(+)(+)$ $\boxed{\text{Roots: } x=-3,-1,3}$

②/③

−		+		−		+
(−4)	−3	(−2)	−1	(0)	3	(4)

④ $x \in (-3,-1) \cup (3,\infty)$