

$$f) y = \tan^3 x = (\tan x)^3$$

$$y' = 3(\tan x)(\sec^2 x)(1)$$

$$y' = 3 \tan x \sec^2 x$$

$$e) y = \tan x^3 \quad \begin{matrix} u = x^3 \\ du = 3x^2 dx \end{matrix}$$

$$y' = \sec^2 x^3 (3x^2)$$

$$y' = 3x^2 \sec^2 x^3$$

$$d) y = \cot^3(1-2x) = [\cot(1-2x)]^3$$

$$y' = 3[\cot(1-2x)]^2 [-\csc^2(1-2x)] [2(1-2x)(-2)]$$

$$y' = 12(1-2x) \cot^2(1-2x) \csc^2(1-2x)$$

$$h) y = (x^3)(\csc x)$$

$$y' = x^3(-\csc x \cot x)(1) + 3x^2 \csc x$$

$$y' = 3x^2 \csc x - x^3 \csc x \cot x$$

$$y' = x^2 \csc x [3 - x \cot x]$$

$$k) y = \frac{1}{\sqrt{(\sec 2x-1)^3}} = (\sec 2x-1)^{-3/2}$$

$$y' = -\frac{3}{2}(\sec 2x-1)^{-5/2} (\sec 2x \tan 2x)(2)$$

$$y' = \frac{-3 \sec 2x \tan 2x}{(\sec 2x-1)^{5/2}} = y' = \frac{-3 \sec 2x \tan 2x}{\sqrt{(\sec 2x-1)^5}}$$

$$b) y = \frac{x^3 \tan x}{\sec x} = x^3 \left(\frac{\sin x}{\cos x} \right) \left(\frac{\cos x}{1} \right)$$

$$y = (x^3)(\sin x)$$

$$y' = x^3(\cos x)(1) + 3x^2 \sin x$$

$$y' = x^2 \cos x + 3x^2 \sin x$$

$$y' = x^2(x \cos x + 3 \sin x)$$

$$\text{Ex: } y = \sin(x^2) \quad u = x^2$$

$$y' = \cos(x^2) \cdot 2x$$

$$y' = 2x \cos(x^2)$$

$$\textcircled{1} \text{ n) } y = \sin(\tan x) \quad u = \tan x$$

$$y' = \cos(\tan x) \cdot \sec^2 x$$

$$y' = \sec^2 x [\cos(\tan x)]$$

$$\text{K) } y = \frac{1}{\sqrt{(\sec 2x - 1)^3}} = \frac{1}{(\sec 2x - 1)^{3/2}} = (\sec 2x - 1)^{-3/2}$$

$$y' = -\frac{3}{2} (\sec 2x - 1)^{-5/2} \cdot \sec 2x \tan 2x \cdot (2)$$

$$y' = \frac{-3 \sec(2x) \tan(2x)}{(\sec 2x - 1)^{5/2}}$$

$$y' = \frac{-3 \sec(2x) \tan(2x)}{\sqrt{(\sec 2x - 1)^5}}$$

$$\textcircled{1} \text{ a) } y = \tan^2(\cos x) = [\tan(\cos x)]^2$$

$$y' = 2[\tan(\cos x)] \cdot \sec^2(\cos x) \cdot (-\sin x)(1)$$

$$y' = -2\sin x [\tan(\cos x)] [\sec^2(\cos x)]$$

$$\text{b) } y = \frac{x^2 \tan x}{\sec x} = x^2 \left(\frac{\sin x}{\cos x} \right) \cdot \left(\frac{\cos x}{1} \right) = x^2 \sin x$$

$$y' = x^2 (\cos x)(1) + 2x (\sin x)$$

$$y' = x^2 \cos x + 2x \sin x$$

$$y' = x [x \cos x + 2 \sin x]$$

Final Review

$$\textcircled{1} \text{ b) } f(x) = \frac{2x-2}{x+3} \quad f(x+h) = \frac{2x+2h-2}{x+h+3}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{(x+3)(x+h+3)(2x+2h-2)}{x+h+3} - \frac{(2x-2)(x+3)(x+h+3)}{x+3}}{h(x+3)(x+h+3)}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{2x^2} + \cancel{2xh} - \cancel{2x} + \cancel{6x} + \cancel{6h} - \cancel{6} - (\cancel{2x^2} + \cancel{2xh} + \cancel{6x} - \cancel{2x} - \cancel{2h} - \cancel{6})}{h(x+3)(x+h+3)}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{8h}}{h(x+3)(x+h+3)} = \frac{8}{(x+3)^2}$$

$$h) y = (x^2)(\csc x)$$

$$y' = (x^2)(-\csc x \cot x)(1) + 2x \csc x$$

$$y' = -x^2 \csc x \cot x + 2x \csc x$$

$$y' = x \csc x (-x \cot x + 2)$$

$$y' = x \csc x (2 - x \cot x)$$

$$u = (1-2x)^2$$

$$du = 2(1-2x)(-2)$$

$$i) y = \cot^3(1-2x)^2 = [\cot(1-2x)^2]^3$$

$$y' = 3[\cot(1-2x)^2]^2 [-\csc^2(1-2x)^2] [2(1-2x)(-2)]$$

$$y' = 3 \cot^2(1-2x)^2 [\csc^2(1-2x)^2] [-4(1-2x)]$$

$$y' = 12(1-2x) \cot^2(1-2x)^2 \csc^2(1-2x)^2$$

$$k) \quad y = \frac{1}{\sqrt{(\sec 2x - 1)^3}} = \frac{1}{(\sec 2x - 1)^{3/2}} = (\sec 2x - 1)^{-3/2}$$

$$y' = \frac{-3}{2} (\sec 2x - 1)^{-5/2} (\sec 2x \tan 2x) \quad (\cancel{2})$$

$$y' = \frac{-3 \sec 2x \tan 2x}{(\sec 2x - 1)^{5/2}} = \frac{-3 \sec 2x \tan 2x}{\sqrt{(\sec 2x - 1)^5}}$$

$$b) \quad y = \frac{x^2 \tan x}{\sec x} = x^2 \cdot \frac{\sin x}{\cos x} \cdot \frac{\cos x}{1} = x^2 \sin x$$

$$y' = x^2 (\cos x)(1) + 2x \sin x$$

$$y' = x^2 \cos x + 2x \sin x$$

$$y' = x [x \cos x + 2 \sin x]$$

$$o) \quad y = \tan^2(\cos x) = [\tan(\cos x)]^2$$

$$u = \cos x \\ du = -\sin x (1)$$

$$y' = 2[\tan(\cos x)][\sec^2(\cos x)][-\sin x(1)]$$

$$y' = -2 \sin x \tan(\cos x) \sec^2(\cos x)$$

⑥ Find the points on the curve $y = \frac{x}{x-1}$ where the tangent line is parallel to the line $x+4y=1$

$$\textcircled{1} \quad x+4y=1$$

$$4y = -x+1$$

$$y = \left(-\frac{1}{4}\right)x + \frac{1}{4}$$

$$m = -\frac{1}{4}$$

$$\textcircled{2} \quad y = \frac{x}{x-1}$$

$$y' = \frac{(x-1)(1) - (x)(1)}{(x-1)^2}$$

$$y' = \frac{-1}{(x-1)^2}$$

$$\textcircled{3} \quad \frac{-1}{(x-1)^2} = -\frac{1}{4}$$

$$-(x-1)^2 = -4$$

$$(x-1)^2 = 4$$

$$x^2 - 2x + 1 = 4$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$\begin{array}{l|l} x-3=0 & x+1=0 \\ x=3 & x=-1 \end{array}$$

$$\textcircled{4} \quad y = \frac{x}{x-1}$$

$$y = \frac{3}{3-1}$$

$$y = \frac{3}{2}$$

$$\boxed{(3, \frac{3}{2})}$$

$$y = \frac{-1}{-1-1}$$

$$y = \frac{1}{2}$$

$$\boxed{(-1, \frac{1}{2})}$$

⑥ Find the point on the curve $y = x\sqrt{x}$ where the tangent line is parallel to the line $6x - y = 4$

$$\textcircled{1} \quad 6x - y = 4$$

$$6x - 4 = y$$

$$y = 6x - 4$$

$$m = 6$$

$$\textcircled{2} \quad y = x\sqrt{x} = x(x)^{1/2} = x^{3/2}$$

$$y' = \frac{3}{2}x^{1/2} = \frac{3\sqrt{x}}{2}$$

$$\textcircled{3} \quad \frac{3\sqrt{x}}{2} = \frac{6}{1}$$

$$3\sqrt{x} = 12$$

$$\sqrt{x} = 4$$

$$x = 16$$

$$\textcircled{4} \quad y = x\sqrt{x}$$

$$y = (16)\sqrt{16}$$

$$y = 64$$

$$(16, 64)$$