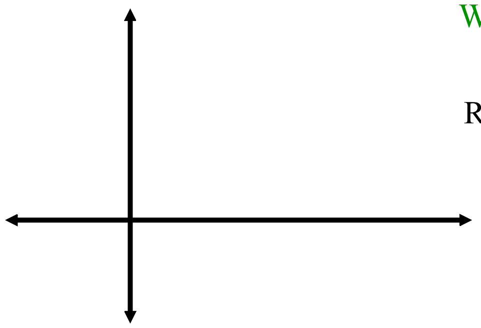
Differentiation Rules

I. Constant Functions

- Sketch the function $y = 2$



What is the slope of the tangent to this graph?

Recall: slope of the tangent is the derivative

The derivative of a constant will always be equal to "0".

Formal Proof:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c - c}{h}$$

$$= \lim_{h \to 0} 0 = 0$$

II. Power Functions

We want to come up with a rule to differentiate functions of the form $f(x) = x^n$, $x \in R$

Here are a couple derivatives that we would have already looked at using limits:

$$\frac{d}{dx}(x^2) = 2x \qquad \frac{d}{dx}(x^3) = 3x^2$$

Using the definition of a derivative to differentiate $f(x) = x^4$ would lead to ...

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h}$$

$$= \lim_{h \to 0} \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h}$$

$$= \lim_{h \to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$$

$$= \lim_{h \to 0} (4x^3 + 6x^2h + 4xh^2 + h^3) = 4x^3$$

Do you see a pattern emerging?

Uncover the POWER RULE

The Power Rule (General Version) If n is any real number, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Let's practice using the power rule...

Differentiate each of the following functions:

1. $f(x) = x^{25}$

$f'(x) = 25x^{24}$

2. $f(x) = x^{-5}$

$f'(x) = -5x^{-6}$

$f'(x) = \frac{-5}{x^6}$

3. $f(x) = \frac{1}{x^{10}}$

$f(x) = x^{-10}$

$f'(x) = -10x^{-11}$

$f'(x) = \frac{-10}{x^{11}}$

4. $f(x) = \sqrt[5]{x^7}$

$f(x) = x^{7/5}$

$f'(x) = \frac{7}{5}x^{2/5}$

$f'(x) = \frac{7\sqrt[2]{x^2}}{5}$

Constant Multiples

- The following formula says that <u>the derivative of a constant multiplied by a function is the constant multiplied by the derivative of the function:</u>

The Constant Multiple Rule If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

EXAMPLE 4

(a) $\frac{d}{dx}(3x^4) = 3\frac{d}{dx}(x^4) = 3(4x^3) = 12x^3$

(b) $\frac{d}{dx}(-x) = \frac{d}{dx}[(-1)x] = (-1)\frac{d}{dx}(x) = -1(1) = -1$

Examples:

1. $f(x) = 4x^3$

$f'(x) = 12x^2$

2. $f(x) = \frac{8}{x^2}$

$f(x) = 8x^{-2}$

$f'(x) = -16x^{-3}$

$f'(x) = \frac{-16}{x^3}$

3. $f(x) = 5x^{\frac{6}{5}}$

$f'(x) = 6x^{1/5}$

$f'(x) = 6\sqrt[5]{x}$

4. $f(x) = (3x^2)^2$

$f(x) = 9x^4$

$f'(x) = 36x^3$

Sums and Differences

- These next rules say that <u>the derivative of a sum (difference) of functions is the sum (difference) of the derivatives</u>:

The Sum Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

The Difference Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

Demonstrate what this all means...

Differentiate each of the following:

1. $f(x) = 2x^4 + \sqrt{x}$

$f(x) = 2x^4 + x^{1/2}$

$f'(x) = 8x^3 + \frac{1}{2}x^{-1/2} = 8x^3 + \frac{1}{2x^{1/2}}$

2. $f(x) = 6x^4 - 5x^3 - 2x + 17$

$f'(x) = 24x^3 - 15x^2 - 2$

3. $f(x) = (2x^3 - 5)^2$

$f(x) = 4x^6 - 20x^3 + 25$

$f'(x) = 24x^5 - 60x^2$

Recall the derivative of a function is equal to the slope of a line that is tangent to the function.

Find the slope of the tangent line to the function at the given "x" coordinate!

$$f(x) = 3x^2 \quad \text{at } x = 4$$

① Find $f'(x)$

$f'(x) = 6x$

② Sub x-value into $f'(x)$

$f'(4) = 6(4)$

$= 24$ ← Slope of tangent line

Example:

Find the equation of the tangent line to the curve $f(x) = x^6$ at the point (-2, 64)

Remember that the equation of a line is found by using the point-slope formula... $y - y_1 = m(x - x_1)$

The curve is the graph of the function $f(x) = x^6$ and we know that the slope of the tangent line at (-2, 64) is the derivative $f'(-2)$

- Find derivative
- Fill in x value and solve for slope
- Use equation of a line formula and solve

$$f(x) = x^6$$

$$f'(x) = 6x^5$$

$$f'(-2) = 6(-2)^5$$

$$= 6(-32)$$

$$= -192$$

$$y - y_1 = m(x - x_1)$$

$$y - 64 = -192(x + 2)$$

$$y - 64 = -192x - 384$$

$$\boxed{192x + y + 320 = 0}$$

Homework

⑤ $f(x) = \frac{1}{x}$ $f(x+h) = \frac{1}{x+h}$

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \cdot \frac{(x)(x+h)}{(x)(x+h)}$$

$$= \lim_{h\to 0} \frac{x - (x+h)}{hx(x+h)}$$

$$= \lim_{h\to 0} \frac{-h}{h(x)(x+h)} = \boxed{\frac{-1}{x^2}}$$