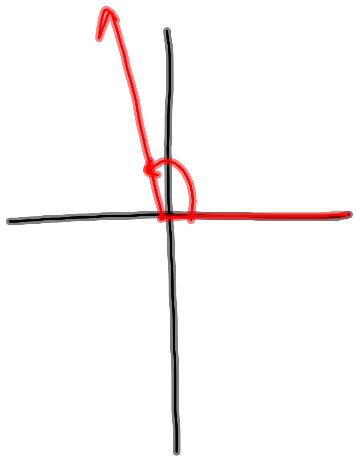
Questions from Homework

2 a) 101°

$101^\circ \times \frac{\pi}{180^\circ}$

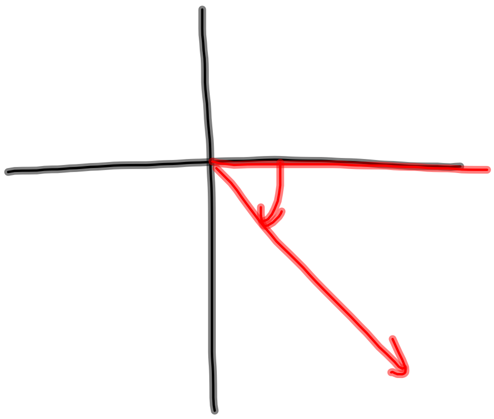
$\frac{101\pi}{180}$

1.76



3 c) $-\frac{2\pi}{9}$

-0.69



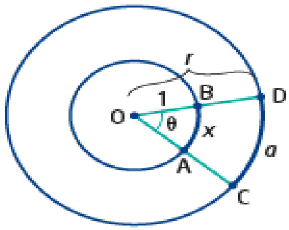
$-\frac{2\cancel{\pi}^{1}}{\cancel{9}_{1}} \times \frac{\cancel{180^\circ}^{20}}{\cancel{\pi}_{1}}$

$\boxed{-40^\circ}$

Arc Length of a Circle

All arcs that subtend a right angle $\left(\frac{\pi}{2}\right)$ have the same central angle, but they have different arc lengths depending on the radius of the circle. The arc length is proportional to the radius. This is true for any central angle and related arc length.

Consider two concentric circles with centre O. The radius of the smaller circle is 1, and the radius of the larger circle is r. A central angle of θ radians is subtended by arc AB on the smaller circle and arc CD on the larger one. You can write the following proportion, where x represents the arc length of the smaller circle and a is the arc length of the larger circle.



$$\frac{a}{x} = \frac{r}{1}$$
$$a = xr \qquad ①$$

Consider the circle with radius 1 and the sector with central angle θ. The ratio of the arc length to the circumference is equal to the ratio of the central angle to one full rotation.

$$\frac{x}{2\pi r} = \frac{\theta}{2\pi}$$ Why is $r = 1$?
$$x = \left(\frac{\theta}{2\pi}\right)2\pi(1)$$
$$x = \theta$$

Substitute $x = \theta$ in ①.

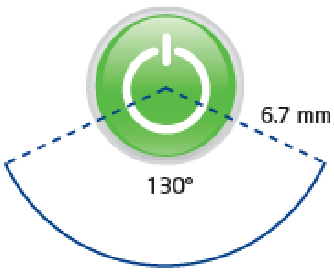
$$\boxed{a = \theta r}$$

This formula, $a = \theta r$, works for any circle, provided that θ is measured in radians and both a and r are measured in the same units.

Determine Arc Length in a Circle

Rosemarie is taking a course in industrial engineering. For an assignment, she is designing the interface of a DVD player. In her plan, she includes a decorative arc below the on/off button. The arc has central angle 130° in a circle with radius 6.7 mm. Determine the length of the arc, to the nearest tenth of a millimetre.

a = arc length
r = radius
θ = radians



$130° \cdot \frac{\pi}{180°}$

$\frac{130\pi}{180}$

$\frac{13\pi}{18}$

$\theta = \frac{13\pi}{18}$

$r = 6.7\text{mm}$

$a = ?$

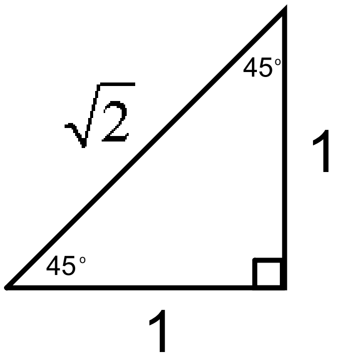
$a = \theta r$

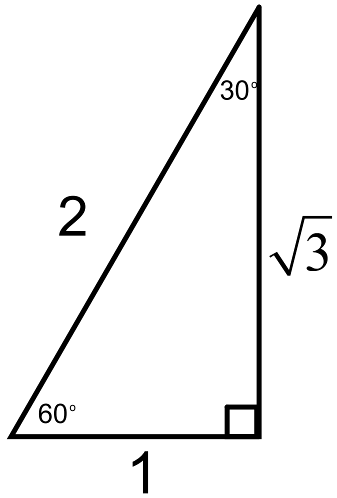
$a = \left(\frac{13\pi}{18}\right)(6.7)$

$\boxed{a = 15.2\text{mm}}$

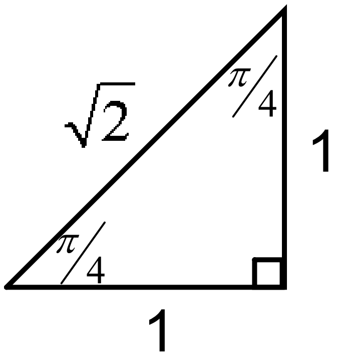
Special Angles

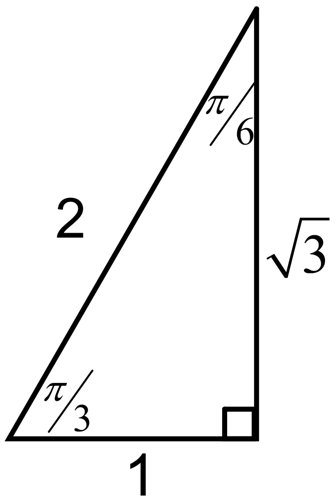
In Degrees:



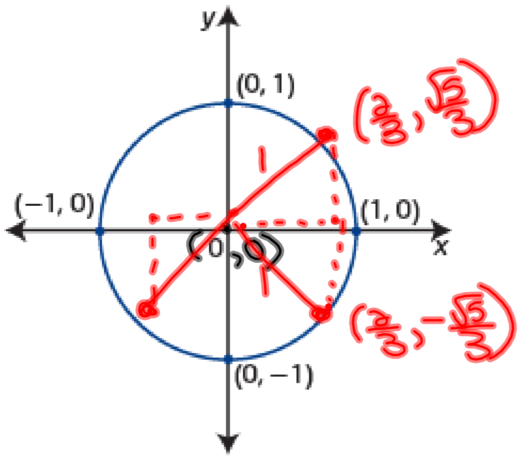


In Radians:





The Unit Circle



unit circle

- a circle with radius 1 unit
- a circle of radius 1 unit with centre at the origin on the Cartesian plane is known as *the* unit circle

The equation of the unit circle is $x^2 + y^2 = 1$.

Determine Coordinates for Points of the Unit Circle

Determine the coordinates for all points on the unit circle that satisfy the conditions given. Draw a diagram in each case.

a) the x-coordinate is $\frac{2}{3}$

b) the y-coordinate is $-\frac{1}{\sqrt{2}}$ and the point is in quadrant III

a) $x = \frac{2}{3}$

$x^2 + y^2 = 1$

$\left(\frac{2}{3}\right)^2 + y^2 = 1$

$\frac{4}{9} + y^2 = 1 \quad -\frac{4}{9}$

$y^2 = \frac{5}{9}$

$y = \pm\sqrt{\frac{5}{9}} = \pm\frac{\sqrt{5}}{\sqrt{9}} = \pm\frac{\sqrt{5}}{3}$

$\left(\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$ and $\left(\frac{2}{3}, -\frac{\sqrt{5}}{3}\right)$

b) $y = -\frac{1}{\sqrt{2}}$

$x^2 + y^2 = 1$

$x^2 + \left(\frac{-1}{\sqrt{2}}\right)^2 = 1$

$x^2 + \frac{1}{2} = 1$

$x^2 = \frac{1}{2}$

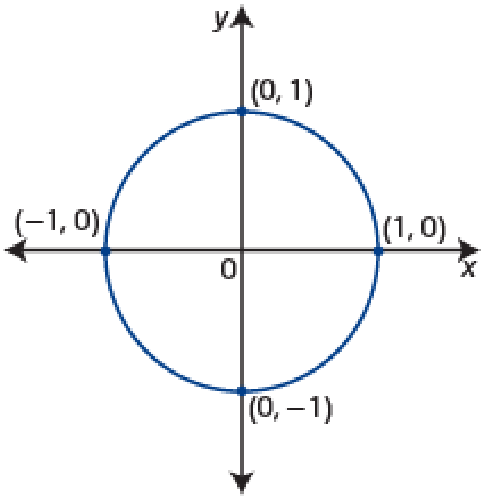
$x = \pm\frac{1}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$

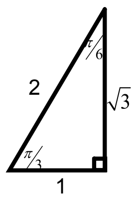
Multiples of $\frac{\pi}{3}$ on the Unit Circle

a) On a diagram of the unit circle, show the integral multiples of $\frac{\pi}{3}$ in the interval $0 \leq \theta \leq 2\pi$.

b) What are the coordinates for each point $P(\theta)$ in part a)?

c) Identify any patterns you see in the coordinates of the points.





a) Multiples $\frac{\pi}{3}$

$0\left(\frac{\pi}{3}\right) = \frac{0\pi}{3} = 0$

$1\left(\frac{\pi}{3}\right) = \frac{\pi}{3}$

$2\left(\frac{\pi}{3}\right) = \frac{2\pi}{3}$

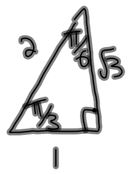
$3\left(\frac{\pi}{3}\right) = \frac{3\pi}{3} = \pi$

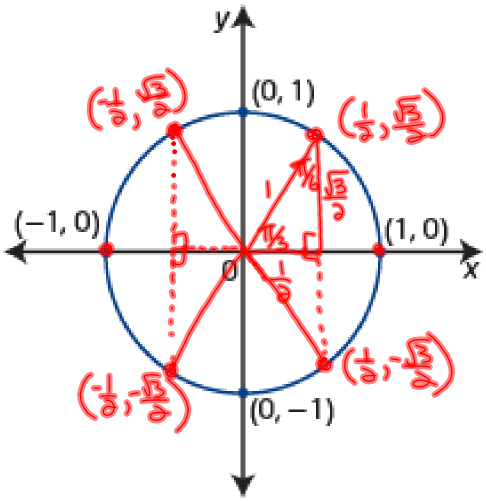
$4\left(\frac{\pi}{3}\right) = \frac{4\pi}{3}$

$5\left(\frac{\pi}{3}\right) = \frac{5\pi}{3}$

$6\left(\frac{\pi}{3}\right) = \frac{6\pi}{3} = 2\pi$

b) In order for this diagram to fit on the unit circle we must scale it back (Remember the unit circle has a radius of 1).

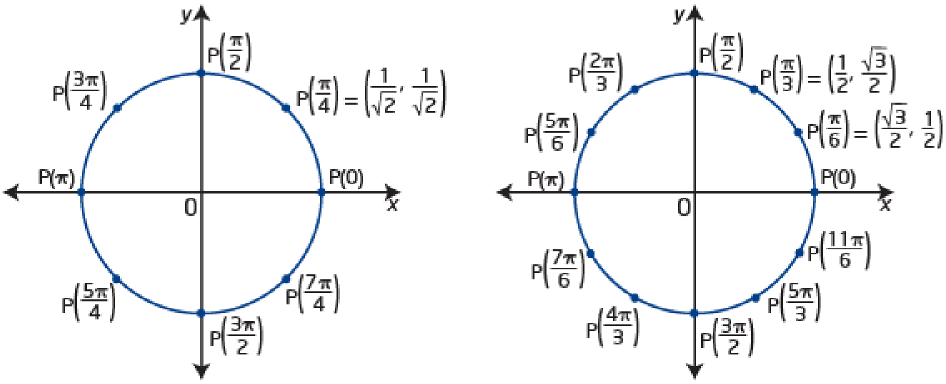
So  becomes 



Key Ideas

- The equation for the unit circle is $x^2 + y^2 = 1$. It can be used to determine whether a point is on the unit circle or to determine the value of one coordinate given the other. The equation for a circle with centre at $(0, 0)$ and radius r is $x^2 + y^2 = r^2$.
- On the unit circle, the measure in radians of the central angle and the arc subtended by that central angle are numerically equivalent.
- Some of the points on the unit circle correspond to exact values of the special angles learned previously.
- You can use patterns to determine coordinates of points. For example, the numerical value of the coordinates of points on the unit circle change to their opposite sign every $\frac{1}{2}$ rotation.

  If $P(\theta) = (a, b)$ is in quadrant I, then both a and b are positive. $P(\theta + \pi)$ is in quadrant III. Its coordinates are $(-a, -b)$, where $a > 0$ and $b > 0$.



Homework

- **Show the integral multiples of $\pi/6$ on the unit circle and list the coordinates for each.**
- **Show the integral multiples of $\pi/4$ on the unit circle and list the coordinates for each.**
- **Workbook #1-6 pages 123-126**